

## 15-1

B

NS $1.5 \bigodot$ Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

## Understanding Integers

What are integers and what situations can
integers represent?

The highest point in Louisiana is Driskill Mountain at five hundred thirty-five feet above sea level. The lowest point is New Orleans at eight feet below sea level. How can you write those highest and lowest points with integers?
 sea level

## Guided Practice*

## Do you know HOW?

In 1 through 4, write an integer for each word description.

1. Ten degrees below zero
2. Seventy degrees above zero
3. Two hundred thirty feet above sea level
4. Fifty-two feet below sea level

## Do you UNDERSTAND?

5. In the example above, what is the opposite elevation of Driskill Mountain, written as an integer?
6. How far away from sea level is 512 feet below sea level?
7. How would you show sea level represented as an integer?

## Independent Practice

In 8 through 12, use the number line to identify the integer at each point.

8. $T$
9. $W$
10. $X$
11. $Y$
12. $Z$

In 13 through 20, write an integer for each word description.
13. A withdrawal of $\$ 20$
15. A gain of three inches
17. A loss of 7 pounds
19. 6 steps forward
14. A deposit of one hundred dollars
16. A loss of six yards
18. A temperature drop of 2 degrees
20. 10 seconds before blastoff

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Distance above sea level is greater than zero. It is represented by a positive integer. ${ }^{+535}$ Distance below sea level is less than zero. It is represented by the negative integer. -8 Integers name magnitude (distance) and direction from 0 .


The magnitude of ${ }^{-} 8$ is 8 . The magnitude of ${ }^{+} 535$ is 535 . The direction is negative. The direction is positive.

Integers are the whole numbers and their opposites; 0 is its own opposite.

Numbers that are opposites of each other have the same magnitude (distance from 0 ).
-5 and +5 are the same distance from 0 .


| Negative integers <br> are less than zero. | Positive integers <br> are greater than zero. |
| :--- | :--- |
| -5 is read | +5 is read |
| "negative five." | "positive five." |

-2 is the opposite of ${ }^{+} 2$.
+4 is the opposite of -4 .

## Problem Solving

21. A football team started at the 20 -yard line. In the first two plays, the team lost 4 yards and gained 4 yards. Where did they end up?
22. A movie company announced that one of its releases lost two million, eight hundred fifty-seven thousand, nine hundred dollars. Write that number in integer form.
23. Number Sense Julie needs to select an integer that is two less than ${ }^{-11}$. What number should she pick? How did you find the number?
24. Think About the Process Pam made $\$ 168.75$ at a craft fair. She sold 75 of the 125 book covers she made. Which expression can you use to find the price she charged for each book cover?

A $\$ 168.75 \div 75$
B $\$ 168.75 \div(125+75)$
C $\$ 168.75 \div 125$
D $\$ 168.75 \div(125-75)$
22. Adam has $1 \frac{1}{2}$ feet of aluminum wire, 1.29 feet of copper wire, and $1 \frac{5}{8}$ feet of steel wire. Adam has the most of which kind of wire?
24. At midnight, the temperature was 2 degrees. It went down 5 degrees, then it went up 3 degrees, and then dropped 2 degrees. What was the final temperature? Show your answer on a number line.
26. The Mariana Trench is located in the floor of the western North Pacific Ocean. It is 35,798 feet below sea level. Express this depth as an integer.
28. Writing to Explain Describe how to find the surface area of the rectangular prism shown below. Then find the surface area.


NS $1.5 \bigodot$ Identify and represent on a number line decimals, fractions, mixed number, and positive and negative integers.

# Comparing and Ordering Integers 

How do you compare and order integers?
Alan's family spent a week at a resort in Utah. The resort newspaper listed the low temperature for each night that week. Which night had a lower temperature, Thursday or Friday? What is the order from least to greatest of the low temperatures?

This week's temperatures ( ${ }^{\circ} \mathrm{F}$ )

|  | Low | High |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Monday | $6^{\circ}$ | $\vdots$ | $30^{\circ}$ |  |
| Tuesday | $\vdots$ | $0^{\circ}$ | $\vdots$ | $28^{\circ}$ |
| Wednesday | $\vdots$ | $4^{\circ}$ | $21^{\circ}$ |  |
| Thursday | $\vdots$ | $-7^{\circ}$ | $17^{\circ}$ |  |
| Friday | $\vdots$ | $-3^{\circ}$ | $19^{\circ}$ |  |

## Guided Practice*

## Do you know HOW?

In 1 through 4, compare. Use $>,<$ or $=$ for


1. $+4 \bigcirc+3$
2. $-2 \bigcirc+2$
3. $-1 \bigcirc-4$
4. $-10 \bigcirc-11$

In 5 and 6, order from least to greatest.
5. $+8,-5,-2$
6. $-10,+2,-3$

## Do you UNDERSTAND?

7. Compare -7 and -3 . Use the $>$ sign.
8. In the example above, if the temperature on Wednesday night was ${ }^{-} 9^{\circ}$ F, which night would have been colder, Wednesday or Thursday?
9. In the example above, order the week's high temperatures from least to greatest.

## Independent Practice

In 10 through 17, compare using $>$, $<$ or $=$ for

10. ${ }^{+} \bigcirc+3$
11. $-4 \bigcirc+9$
12. $+5 \bigcirc-2$
13. $-11 \bigcirc-10$
14. $-8 \bigcirc-15$
15. $+10 \bigcirc+11$
16. $-7 \bigcirc-6$
17. $-1 \bigcirc 0$

In 18 through 33, order from least to greatest.
18. ${ }^{+} 1,-7,-5$
19. $0,-3,+6$
20. $-5,{ }^{+} 10,-1$
21. $-4,+11,-6$
22. $0,+8,-8$
23. $+3,+1,+5$
24. $-2,-8,-1$
25. $-23,-50,-42$
26. $+15,-5,+6,-2$
27. $-20,-1,-9,-13$
28. $0,-19,+5,-4$
29. $-5,-20,-10,-15$
30. $+6,-3,-2,+7$
31. $-18,-3,+3,-8$
32. $-5,0,+1,-20$
33. $-6,-7,-8,-9$

Compare the integers -7 and -3 .
Locate - 7 and - 3 on a number line. Integers, just like whole numbers, fractions, and decimals, increase in value as you move from left to right.


The integer -7 is farther to the left on a number line than -3 . So, -7 is less.
$-7<-3$

Order the integers 6, 0, 4, -7, and -3. Locate the numbers on a number line.


From least to greatest, the week's low temperatures are $-7,-3,0,4$, and 6.

Positive numbers are often written without the ${ }^{+}$sign.

Thursday night was colder than Friday night.

## Problem Solving

34. Sam's class played a history game. Team A had a score of 200, Team B had a score of ${ }^{-300}$, Team C had a score of ${ }^{-1} 100$ and Team D had a score of 500 . Order the scores, from greatest to least.
35. Estimation What is a good estimate for the value of point $C$ on the number line?

36. Algebra Write an integer for $x$ to make each statement true.
a $x>-3$
b $x<{ }^{+1}$
c $-13<x$
37. Reasoning A number, $x$, is four units to the left of -5 on the number line. What is the value of $x$ ? Is $x$ greater than or less than -5 ?
38. Number Sense Which integer is neither positive nor negative?

In 40 through 42, use the map at the right.
40. Of the states shown, which had the lowest record temperature?
41. Which state had the warmest record low temperature?
42. List the record low temperatures in order from least to greatest.

NS $1.5 \bigodot$ Identify and represent on a number line decimals, fractions, mixed number, and positive and negative integers.

## Integers and the Number Line

How can you name and plot integers, fractions, and decimals on the same number line?
Some points are plotted on the number line below. Write a number to name each of the points $A, Z, D$, and $W$. Use integers, fractions, mixed numbers, or decimals. Name the points that represent 2.38 , and $1 \frac{1}{4}$, and ${ }^{-1}$.


## Guided Practice*

## Do you know HOW?

In 1 through 4, use the number line below. Write the number for each point.


1. $A$
2. $B$
3. $C$
4. $D$

In 5 and 6, show each set of numbers on a number line.
5. $1 \frac{1}{4}, 0.75,-1,-2$
6. $-2,1 \frac{3}{4}, 0.5,0$

## Do you UNDERSTAND?

7. In the example above, how could you find the points for fractions?
8. In the example above, describe where you would plot ${ }^{+1.35}$.
9. What number is represented by point $B$ above?
10. Which point above represents $2 \frac{3}{4}$ ?

## Independent Practice

In 11 through 20, use the number line below. Write the number for each point.

11. $W$
12. $B$
13. $E$
14. $D$
15. $A$
16. $Z$
17. $Y$
18. $C$
19. $V$
20. $X$

In 21 through 23, order the numbers in each set from least to greatest.
Show each set of numbers on a number line.
21. $-2,1, \frac{1}{3}, 1 \frac{2}{3}$
22. $3,-3,1.5,2 \frac{3}{4}$
23. $0.5,1 \frac{3}{8}, 2.25,-1$

## Write a number to name a point.

The distance between the integers 0 and 1 is divided into 10 equal parts.
Point $A$ is to the left of zero. Point $A$ is at ${ }^{-3}$.
Point $Z$ is to the right of zero. Point $Z$ is at ${ }^{+} 3$.
Point $D$ is at 0.4 or $\frac{4}{10}$.
Point $W$ is halfway between 1.7 and 1.8.
So, point $W$ is at 1.75 or $1 \frac{3}{4}$.

Find which point represents a given number.

Since 2.38 is between 2.3 and 2.4 and is closer to 2.4 , point $X$ represents 2.38 .

Since $1 \frac{1}{4}=1.25$ and 1.25 is halfway between 1.2 and 1.3 , point $E$ represents 1.25 .

Since ${ }^{-1}$ is opposite of ${ }^{+} 1$, point $C$ represents ${ }^{-1}$.

## Problem Solving

24. Writing to Explain What might you do to order the numbers $-8,-10,+8$, and ${ }^{+} 5$ without using a number line?
25. Geometry Find the perimeter of the equilateral triangle below.

26. Which number in the following choices makes the statement true?

$$
-6>n
$$

A 0
B -5
C -7
D -4
30. The table below names the lowest points on each continent. Order the points from lowest to highest.

| Continent | Height (feet) |  |
| :--- | :---: | :---: |
| Africa | $\vdots$ | -512 |
| Antarctica | $\vdots$ | $-8,327$ |
| Asia | $\vdots$ | $-1,348$ |
| Europe | $\vdots$ | -92 |
| North America | -282 |  |
| Oceania | $\vdots$ | -52 |
| South America | -131 |  |

25. Number Sense Kathy owes Marty $\frac{1}{2}$ of a dollar. Mina owes Marty 0.6 of a dollar. Who owes Marty more money?
26. The larva of the Colorado Potato Beetle is 0.5 inches long. The adult Colorado Potato Beetle is $\frac{3}{8}$ inch long. Which one is smaller, the adult or the larva?
27. Joey got 19 out of 23 questions correct on his Social Studies test. If each question was worth 3 points, how many points did Joey earn?
28. Which number line shows a point that best represents ${ }^{+1.6 \text { ? }}$

A
$\begin{array}{llllllllll}\mathbf{B} & -1 & -2 & -1 & 0 & +1 & +2 & +3\end{array}$
C

$\mathbf{D} \begin{array}{cccccccc}-3 & -2 & -1 & 0 & +1 & +2 & +3\end{array}$

## Adding Integers

 How can you add two integers?A football coach keeps a record of the yards gained or lost on each play. What was the total number of yards gained or lost after the two plays shown?
Choose an Operation Add to find the result of joining the two plays.


## Another Example How can you add two negative integers?

You know how to represent integers on a number line. In this lesson you are using a number line to add integers.

On the next play, the football team lost 4 yards and received a 5 -yard penalty. How many yards did the team lose on that play?

Find $-4+{ }^{-5}$.
Start at 0. Face the positive integers. Then move backward 5 steps for ${ }^{-5}$. Move backward 4 steps for ${ }^{-4}$.


You stop at ${ }^{-9}$. So, $-4+-5=-9$
The football team lost a total of 9 yards on that play.
When adding two integers with the same sign, you move in the same direction on the number line, so you add the magnitude of the numbers. You move in the same direction, so the sign of the sum will be the same as the sign of the addends.

## Explain 11

1. In the example above, why did you move backwards twice?
2. How would the sum be different if the team had gained 4 yards and the 5 -yard penalty was against the other team? How would that sum look on the number line?

Rules for adding integers on a number line:

Start at 0; face the positive integers.

Walk forward for positive integers.
Walk backward for negative integers.

Find ${ }^{+7}+{ }^{-10}$.
Walk forward 7 steps for ${ }^{+} 7$. Then walk backward 10 steps for ${ }^{-10}$.


To add two integers with different signs you move in different directions on the number line, so subtract the magnitudes of the numbers. You move farther in the direction of the number with the greater magnitude, so the sign of the sum will be the sign of that number.
${ }^{+} 7+{ }^{-1} 10={ }^{-3}$ After the two plays, the team lost 3 yards.

## Guided Practice*

## Do you know HOW?

In 1 through 8, use a number line to find each sum.

1. ${ }^{+} 2+-7$
2. ${ }^{+} 4++3$
3. $-3+-1$
4. $+5+-9$
5. $+5+-3$
6. $-7++9$
7. $-4+{ }^{+}$
8. $-6+-3$

## Do you UNDERSTAND?

9. Number Sense The integers ${ }^{+} 4$ and - 4 are opposites. What statement can you make about the sum of any integer and its opposite?
10. The football team gained 12 yards on the first play and then lost 9 yards on the next play. How many yards were gained or lost after the two plays?

## Independent Practice

In 11 through 31, use a number line to find each sum.
11. ${ }^{+} 2++3$
12. $-5++4$
13. $+6+-4$
14. $-8+-5$
15. $0+-4$
16. $+7+-5$
17. $-9++2$
18. $+7+-6$
19. $-8+-3$
20. ${ }^{+} 7+{ }^{+} 9$
21. $-4+{ }^{+} 8$
22. $-9+-5$
23. $-6+-5$
24. $-8++7$
25. $-7+-7$
26. $-6+{ }^{+} 8$
27. $-9+-8$
28. ${ }^{+} 9+{ }^{-12}$
29. $-11+-6$
30. $+14+-5$
31. ${ }^{+} 1+{ }^{-7}+{ }^{+} 12+{ }^{-1}+{ }^{+} 7$
32. In 31, how can you find the sum without using a number line?
33. Number Sense Mrs. Gomez finds her checking account balance after each transaction. A copy of her check register is shown at the right.
a Would a check be represented by a positive or a negative integer?
b Would a deposit be represented by a positive or a negative integer?
c Find the checking account balance after each transaction. On which day was the balance the greatest? the least?
d What is the ending balance?
35. Xavier did math homework for $\frac{3}{4}$ hour, reading homework for $\frac{2}{3}$ hour, and science homework for $\frac{1}{3}$ hour. How much time did Xavier spend doing homework?
37. Algebra Use a number line to find each missing value.
a $-4+\square=9$
b $9+\square=-1$
c $\quad+3=0$
d $-7+\square={ }^{-13}$
39. The sum of Eric's and Gus's heights is $135 \frac{3}{4}$ inches. Eric's height is 68.5 inches. How tall is Gus?
41. What is the prime factorization of 18 ?
42. Geometry Marcy is making a quilt and cut a triangle from a piece of fabric. What is the measure of the third angle of Marcy's triangle?


Find each sum.

1. $-14+{ }^{+} 10$
2. ${ }^{+} 8+(-6)$
3. $-8+(-12)$
4. ${ }^{+} 9+(-3)$
5. $-5+(-5)$
6. $-7+{ }^{+} 8$

Find each difference. Simplify if possible.
7. $3 \frac{1}{2}$
8. $4 \frac{5}{12}$
9. $5 \frac{3}{8}$
10. 4
$-1 \frac{3}{8}$
$-4 \frac{1}{4}$
$\begin{array}{r}-\quad 3 \\ \hline\end{array}$
$-2 \frac{3}{10}$
11. $5 \frac{5}{6}$

| $-4 \frac{11}{12}$ |
| :--- |

12. $1 \frac{4}{5}$
13. $3 \frac{2}{3}$
$-1 \frac{1}{6}$
14. $6 \frac{7}{8}$
$-4 \frac{5}{8}$
15. $4 \frac{1}{3}$ $-2 \frac{2}{3}$

Error Search Find each sum or difference that is not correct. Write it correctly and explain the error.
16. $-6++4=2$
17. $-3+(-8)=-11$
18. $5 \frac{4}{9}$
$-3 \frac{2}{3}$
19. 5
$-1 \frac{3}{5}$

## Number Sense

Estimating and Reasoning Write whether each statement is true or false.
Explain your reasoning.
20. If $a<0$ and $b<0$, then $a+b$ is negative.
21. The difference of 29.13 and 17.95 is greater than 11 and less than 13.
22. The sum of $\frac{3}{17}$ and $\frac{5}{17}$ can be simplified.
23. The quotient of $3,746 \div 50$ has a remainder less than 50 .
24. The expression $w-2.5$ equals 5.5 when $w=3$.
25. The product of 5 and 2.5 is less than the product of 2 and 5.5.

NS 2.1 C_m Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

## Subtracting Integers

 How can you subtract integers?On a winter day, George checked the temperature during the afternoon and again at night. How many degrees did the temperature drop?
Choose an Operation Subtract to find the temperature change.

## ${ }^{\circ} \mathrm{C}$.

 Guided Practice*
## Do you know HOW?

In 1 through 6, rewrite each subtraction problem using addition. Then find the answer. Use a number line to check.

1. $-1-+3$
2. $-9-+4$
3. $+8-+5$
4. $+4-+10$
5. $-6-+3$
6. ${ }^{-10}-{ }^{+} 1$

## Do you UNDERSTAND?

7. Reasoning Why is subtracting an integer the same as adding its opposite?
8. In the example above, suppose the afternoon temperature had been ${ }^{+} 10^{\circ} \mathrm{F}$ and the night temperature had been ${ }^{-} 4^{\circ}$ F. How many degrees did the temperature drop?

## Independent Practice

In 9 through 32, rewrite each subtraction problem using addition.
Then find each answer. Use a number line to check.
9. $-1-+2$
10. $-8-+3$
11. $-6-+8$
12. ${ }^{+} 3-+12$
13. $0-+5$
14. ${ }^{+} 5-+6$
15. $-3-+10$
16. $-7-+11$
17. $-10-+5$
18. $-12-+1$
19. $-7-+3$
20. ${ }^{+} 6-+3$
21. ${ }^{+} 8-+12$
22. $-4-{ }^{+} 8$
23. $-5-+2$
24. $-3-+6$
25. $-5-+1$
25. $-2-+2$
27. $+5-+7$
28. $-9-{ }^{+} 8$
29. $+7-+9$
30. $-5-{ }^{+} 8$
31. $-9-+6$
32. ${ }^{+} 8-+11$

Find - 3 - + 6 .
Start at 0. Face the positive integers. Walk backward 3 steps
for ${ }^{-3}$.


The subtraction sign (-) means turn around.

Then walk forward 6 steps for ${ }^{+} 6$.


So, $-3-+6=-9$
$-3-+6$ can be thought of as
-3 plus the opposite of ${ }^{+} 6$.
$-3-+6=-3+-6=-9$
The temperature dropped $9^{\circ} \mathrm{F}$.

## Problem Solving

33. The temperature rose from ${ }^{-} 10^{\circ} \mathrm{F}$ to ${ }^{+} 15^{\circ}$. How much did the temperature rise?
34. Lincoln Elementary students sold school mugs for a fundraiser. They bought the mugs for $\$ 3$ each and sold them for $\$ 5$ each. If the students sold 248 mugs, how much money did they raise?
35. Owen has a charge account at the school store. His mom deposited $\$ 10$ in the account at the beginning of the school year. What is the balance after he purchased the items listed at the right?
36. Algebra Write the integers that could replace $x$ to make each statement true.
a $-6<x<+2$
b $+6>x>-2$
37. Writing to Explain Describe the steps to find ${ }^{+} 4-+7$ on a number line.
38. Susie cut a blueberry pie into slices. Each slice was $\frac{1}{8}$ of the pie. How many slices of pie were there?
39. The highest elevation in North America is Mt. McKinley, Alaska, at 20,320 feet. The lowest elevation is -282 feet in Death Valley, California. What is the difference between the two elevations?

| Transaction | $\vdots$ | Amount |
| :--- | :---: | :---: |
| Pencils | $\vdots$ | $\$ 1.50$ |
| Notebook paper | $\vdots$ | $\$ 0.89$ |
| Protractor | $\vdots$ | $\$ 1.29$ |
| Pocket dictionary | $\vdots$ | $\$ 4.49$ |
| Trail Mix | $\vdots$ | $\$ 1.19$ |

39. Number Sense The area of California is about 163,696 square miles. What is that measurement rounded to ten thousand square miles?
A 200,000
C 164,000
B 163,000
D 160,000
40. Finish this sentence "Subtracting ${ }^{+} 10$ is the same as adding ? ."

## Simplifying Expressions <br> How can you evaluate an algebraic expression with integers?

The rules of the game Number Signs penalize a player for taking too much time. Greg has a score of 12 points down. What will be his score if he takes too much time on his next turn?

Choose an Operation Subtract to find a score after losing points.

Another Example How do you evaluate an expression with more than one variable?
When more than one variable is involved, replace each variable with a number. If only addition and subtraction are involved, proceed from left to right, grouping two numbers at a time.

Evaluate $a+b-c$ for $a=-8, b=-6$ and $c=+12$.

$$
\begin{aligned}
a+b-c & =-8+-6-+12 & & \text { Replace } a \text { with }-8, b \text { with }-6 \text { and } c \text { with }+12 \\
& =-14-+12 & & \text { Add }-8+-6 . \\
& =-14+{ }^{+} 12 & & \text { Rewrite the subtraction as an addition of the opposite. } \\
& =-26 & & \text { Add }-14+{ }^{-} 12 .
\end{aligned}
$$

## Guided Practice*

## Do you know HOW?

In 1 through 4, evaluate each expression for $n=-5$ and $n=+7$.

1. $n+{ }^{+} 5$
2. $n-{ }^{+} 9$
3. $n-+5$
4. ${ }^{+} 8+n$

In 5 and 6, evaluate each expression for $r={ }^{-9}, s=-6$, and $t={ }^{+} 8$.
5. $r+s+t$
6. $s-t+r$

## Do you UNDERSTAND?

7. a In the example above, what was the opposite operation used to evaluate the expression?
b Use that operation to rewrite the expression in the example.
8. a Write an expression to show what Greg's score would have been if the penalty for too much time was to lose 10 points?
b What would his score have been?

Since 5 points are subtracted from the player's score, s, the expression $s-5$ shows how to find a player's score after getting the time penalty.

Greg's score of 12 points down can be expressed with the integer ${ }^{-12}$.

Evaluate $s-+5$ for $s={ }^{-12}$.
$s-+5=-12-+5$ Replace $s$ with -12 .
$=-12+-5$ Subtracting an integer is the same as adding its opposite.
$=-17 \quad$ Add -12 and -5.
Greg's score will be ${ }^{-17}$, or 17 points down if he takes too much time on his next turn.

## Independent Practice

In 9 through 12, evaluate each expression for $x={ }^{+} 6$ and $x={ }^{-} 8$
9. $x-+3$
10. $+4+x$
11. $x+{ }^{-9}$
12. ${ }^{+} 12+x$

In 13 through 20, evaluate each expression for $a=-4, b=+2$ and $c=-10$
13. $a-{ }^{+}+13$
14. $c-{ }^{+}+15$
15. $b-+2$
16. $-18+a$
17. $c-b$
18. $a+b+c$
19. $a-b+c$
20. $c-b+a$

## Problem Solving

21. Which expression names the location of a scuba diver who started at -12 feet and then moved down 3 feet?

A $-12++3$
B +3--12
C $-12+-3$
D $-3+-12$
23. Reasoning The temperature was ${ }^{-} 9^{\circ} \mathrm{F}$ at 6:00 A.м. Write an expression to name the temperature at 3:00 p.m. after it rose $15^{\circ} \mathrm{F}$.
25. California was the 31 st state admitted to the Union. The expression $1787+c$ represents the year California obtained statehood. If $c=63$, what year did California become a state?
22. Cory uses red paint and blue paint to make purple paint. How much red paint did he use if he made 9 quarts of purple?

24. Barry's CD shelf is 36 inches long. If each $C D$ is $\frac{3}{8}$ of an inch wide, how many CDs can Barry put on one shelf?
26. Writing to Explain Jerome says that if you use the LCD when subtracting fractions, you never have to simplify the answer. Do you agree? Why or why not?

MR 2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. Also MR 2.0, NS 2.0

## Problem Solving

## Work Backward

Arnie, Brad, Caren, and Danica sold nature photographs to raise money for their hiking club. Brad raised twice as much money as Arnie. Caren raised \$100 more than Brad, and Danica raised half as much as Caren. How much money did each person raise?


## Guided Practice*

## Do you know HOW?

You can solve this problem by working backward. Check your work.

1. The Penguins' hockey practice ended at 7:00 p.m. The team began practice by stretching for $\frac{1}{4}$ hour. Then they practiced skating and shooting for $\frac{1}{2}$ hour. During the last $\frac{3}{4}$ hour, the team played a scrimmage game. What time did practice start?

## Do you UNDERSTAND?

2. Writing to Explain Describe what you did to check your solution to Problem 1.
3. In the problem above, why is Danica's $\$ 110$ multiplied by 2 to find the amount that Caren raised?
4. Write a Problem Write a real-world problem that you can solve by working backward.

## Independent Practice

Solve.
5. On a winter night, the temperature dropped $15^{\circ} \mathrm{F}$ between midnight and 6:00 А.м. By 11:00 А.м., the temperature had gone up $7^{\circ}$ F. By 3:00 P.M. the temperature went up another $9^{\circ} \mathrm{F}$, making the temperature $25^{\circ} \mathrm{F}$. What was the temperature at midnight?
6. Mel spent $\$ 9$ at the movies, earned $\$ 24$ mowing lawns, and bought a magazine for $\$ 5$. He had $\$ 21$ left. How much money did he have at the start?

- What do I know?
- What am I asked to find?
- What diagram can I use to help understand the problem?
- Can I use addition, subtraction, multiplication, or division?
- Is all of my work correct?
- Did I answer the right question?
- Is my answer reasonable?

I know:
Danica raised \$110.
Danica raised half as much as Caren.

Caren raised \$100 more than Brad.

Brad raised twice as much money as Arnie.

I know how much each person made compared to someone else.

I can start with the amount Danica raised and work backward.


Caren raised $2 \times \$ 110=\$ 220$.
Brad raised $\$ 220-100=\$ 120$. Arnie raised $\$ 120 \div 2=\$ 60$.
7. The numbers show how many shells in each drawer. Meg has a total of 156 shells. She organizes them by size. How many shells are in drawer 1?
9. A baby gains about $2 \frac{1}{5}$ pounds each month for the first three months after birth. When he was 3 months old, Tyler weighed $14 \frac{1}{10}$ pounds. About how much did Tyler weigh at birth?
11. Geometry What is the area of a square garden with a side that measures 18 feet?
12. Reasoning Donna, Pam, and Mike worked at a school car wash. Donna washed half as many cars as Mike did. Pam washed 9 more than Donna. Mike washed 5 fewer than Pam. If Mike washed 8 cars, how many cars did Pam and Donna wash? What was the total number of cars washed?

10. Briana has $1 \frac{1}{4}$ cups of sesame seeds left in the bag she bought for baking. She used the sesame seeds to make muffins, bread, and bagels to sell at a bake sale. How many cups of sesame seeds were in the bag she bought?

13. Workers need 6 weeks to resurface 15 miles of road. They resurfaced $2 \frac{1}{2}$ miles the fourth week, 3 miles the fifth week, and $4 \frac{1}{2}$ miles the sixth week. How many miles did they resurface during the first three weeks?
A 5 miles
B 10 miles
C 6 miles
D 11 miles

1. The lowest temperature ever recorded in the United States was in Alaska in 1971. It was about ${ }^{-} 80^{\circ}$ Fahrenheit. What is the opposite of -80 ? (15-1)

A -80
B -79
C ${ }^{+} 80$
D ${ }^{+} 81$
2. The elevations of some points of interest are given in the table. These elevations are above, at, or below sea level. Which of the following lists the elevations from least to greatest? (15-2)

| Location | $\vdots$ |
| :--- | :---: |
| Elevation (ft) |  |
| Potomac River | $\vdots$ |
| New Orleans | $\vdots$ |
| Delaware River | -8 |
| Lake Champlain | 0 |

A - $8,0,{ }^{+} 1,{ }^{+} 95$
B $0,{ }^{+} 1,-8,+95$
C $-8,+1,0,+95$
D ${ }^{+} 95,-8,+1,0$
3. What is the value of $5-n$ when $n=+7$ ? (15-6)

A - 12
B -2
C ${ }^{+}$
D ${ }^{+12}$
4. Which of the following can be used to represent a deposit of \$132? (15-1)

A ${ }^{+132}$
B ${ }^{+1}$
C 0
D - 132
5. Which comparison is true? (15-2)
$-15-10-50+5+10+15$
A $-12<-6$
B $-12>-6$
C $-8>-4$
D $+3<-3$
6. Which number line shows

Point $F$ at $1 \frac{4}{5}$ ? (15-3)

B


C


D $F$

7. What is ${ }^{+} 11+(-18)$ ? (15-4)

A ${ }^{+} 29$
B ${ }^{+7}$
C - 7
D - 29
8. What is the integer at Point $L$ ? (15-1)


A - 6
B -5
C -4
D -3
9. What is $+2+(-8)$ ? (15-4)

A ${ }^{-10}$
B -6
C ${ }^{+} 6$
D ${ }^{+10}$
10. Danny had a $-\$ 30$ balance for an amount borrowed from his parents. Then he borrowed \$20 more. What was his balance then? (15-5)

A $\$ 50$
B $\$ 10$
C $-\$ 10$
D - $\$ 50$
11. After a fundraising dinner, a charity has a balance of $\$ 2,530$. They spent $\$ 700$ to host the dinner. If they made $\$ 1,400$ on the event and another \$300 afterwards from a private donation, how much money did the charity have before hosting the dinner? (15-7)

A $\$ 130$
B $\$ 1,530$
C $\$ 2,130$
D \$3,530
12. Which of the following is equal to -6-(+3) (15-5)

A $-6+(+3)$
B ${ }^{+} 6+(+3)$
C ${ }^{+} 6+(-3)$
D $-6+(-3)$
13. What numbers are represented by Points $R$ and $T$ on the number line? (15-3)


A -7 and 8.5
B -8 and 8.5
C -7 and 9.5
D -8 and 9.5
14. On a winter morning, the outside temperature was ${ }^{-} 6^{\circ}$. By noon, it had risen 15 degrees. What was the temperature in the afternoon? (15-4)

A ${ }^{-9}$
B $9^{\circ}$
C $15^{\circ}$
D $21^{\circ}$
15. What is the value of $a+b-c$ when $a=-3, b=+7$ and $c=+1$ ? (15-6)

A - 11
B -10
C +3
D ${ }^{+5}$

Set A, pages 342-343

Write an integer for each point.


Point $A$ is three units from zero and to the left of zero. Point $A$ is at ${ }^{-3}$.

Point $B$ is two units from zero and is to the right of zero. Point $B$ is at ${ }^{+} 2$.

Remember that the + and - signs name a direction from zero.

Write an integer for each description.

1. Two degrees below zero.
2. Fifty-seven feet above sea level.
3. A loss of three yards.

Write an integer for each point.

4. C
5. $E$
6. $B$
7. $H$
8. $D$
9. $A$

Set B, pages 344-345

Compare - 4 and ${ }^{+} 3$. Use $>,<$ or $=$.
Plot the numbers on a number line.


As you move to the right from any point on a number line, the numbers increase in value.
$+3>-4 \quad+3$ is to the right of -4 .
$-4<+3-4$ is to the left of ${ }^{+} 3$.

Remember that numbers increase in value as you move to the right on a number line, and decrease as you move to the left.
Compare. Use $<,>$, or $=$ for each $\bigcirc$.

1. $+3 \bigcirc-3$
2. $-8 \bigcirc+2$
3. $-9 \bigcirc-7$
4. $+3 \bigcirc+2$
5. ${ }^{+} \bigcirc \bigcirc-16$
6. $-14 \bigcirc-10$

Set C, pages 346-347

Order $4, \frac{1}{4},-2$, and 0.5 from least to greatest. $\frac{1}{4}$ is the same as 0.25 . $\quad \frac{1}{2}$ is the same as 0.5 .
Plot the numbers on a number line.


The order from least to greatest is $-2, \frac{1}{4^{\prime}} 0.5,4$

Remember to convert fractions to decimals or decimals to fractions to make the comparing easier.

Order the numbers from least to greatest.

1. $-1, \frac{1}{2},-4,1.25$
2. $4,2.5,-2, \frac{1}{4}$
3. $3.25, \frac{3}{4}, 3,-1$
4. $2 \frac{1}{4},-3,0.68,-2$

Set D, pages 348-350, 352-353

Use a number line to find ${ }^{+} 2+{ }^{-5}$.


Start at 0 and face the positive integers.
Walk forward 2 steps for ${ }^{+} 2$.
Then walk backward 5 steps for ${ }^{-5}$.
You stop at ${ }^{-3}$.
So, $+2+-5=-3$
Find -2 - +10 .
To subtract an integer, add the opposite of the integer being subtracted.
$\begin{aligned}-2-+10 \longrightarrow & -2+{ }^{-10} \\ & -2+{ }^{-10}=-12\end{aligned}$

Remember to move to the right on a number line when adding positive integers and move to the left on a number line when adding negative integers.

1. $+6+-4$
2. $-7+-2$
3. $-8++2$
4. $+10+-5$
5. $-12++3$
6. $-3+-9$

Remember when using a number line to subtract integers, the subtraction sign means to turn around.
7. $+5-+8$
8. $-3-+2$
9. $-5-+8$
10. $-2-+4$
11. ${ }^{+} 7-+9$
12. $0-+6$

Set E, pages 354-355

Find $a+b-c$ for $a=-4, b=+3$ and $c=+5$. $\begin{aligned} a+b-c & =-4+{ }^{+} 3-{ }^{+} 5 & & \text { Replace the variables. } \\ & =-1 & & \text { R } \\ & =-1 & & \text { Add }-4+{ }^{+} 3 . \\ & & & \begin{array}{l}\text { Rewrite subtraction } \\ \text { as addition of the } \\ \text { opposite. }\end{array} \\ & =-6 & & \text { Add }-1+{ }^{-5}\end{aligned}$

Remember to replace the variable with the given values.

Evaluate each expression for $x=+3$
and $x=-2$

1. $+3+x$
2. $x-{ }^{+} 5$
3. $x+{ }^{+}+10$
4. $x-{ }^{+} 1$
5. $x-+3$
6. $x+{ }^{+} 4$

Set F, pages 356-357

To work backward, follow these steps.
Step 1 Identify the unknown initial amount.
Step 2 List each change, starting with the initial amount.

## Step 3

Start at the end result. Work backward using the inverse of each change.

Remember addition and subtraction undo each other.

1. Jean spent $\frac{2}{3}$ hour on math homework and $\frac{1}{2}$ hour on English. Then she spent $\frac{3}{4}$ hour baking. If she finished baking at 8:00, what time did she start doing her homework?
