3

In 2006, 13,000 rubber ducks raced in the 2006 Rubber Duck Derby in Lake Lanier, Georgia. Other locations across the U.S. had derbies that same year. Which derby had more ducks than the one in Georgia? You will find out in Lesson 19-1.

4

California has some of the most popular beaches for surfing in the world. How many different combinations of a wetsuit and a surfboard could a surfer have? You will find out in Lesson 19-7.

Review What You Know!

Vocabulary

Choose the best term from the box.

- axis point
- ordered pair
- 1. A(n) <u>?</u> is either of two perpendicular lines in a graph.
- **2.** A(n) <u>?</u> is an exact location in space.
- **3.** A pair of numbers used to locate a point on a graph is called $a(n) \xrightarrow{?}$.

Number Lines

Use the number line to answer **4** through **6**.

- - 4. Which point is halfway between 1 and 2?
 - **5.** Which point is closest to 1 than to 2?
 - **6.** Which point is closest to 2 than to 1?

Writing Fractions

- 7. If 3 out of 6 marbles are red, what fraction, in simplest form, names the red marbles?
- 8. If 1 out of 5 apples is green, what fraction names the part of the apples that are NOT green?

Division Patterns

Writing to Explain Write an answer to the question.

9. How is the number of zeros in the quotient of 45,000 ÷ 9 related to the number of zeros in the dividend?



SDAP 1.2 Organize and display singlevariable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets. Also SDAP 1.3.

Bar Graphs and Picture Graphs

How do you display data collected from a count or measure?

Students were surveyed about what they do after school. The results were displayed in a bar graph.

A bar graph uses rectangles (bars) to show and compare data that tells how many or how much.

	6
After-Scho	ool Activities
Sports	₩₩₩
Homework	JHT
Chores	₩1
Other	HT HT

Mencs-On

grid paper

Another Example How can you make and interpret picture graphs?

Sonya gathered data about the number of ducks in some of the 2006 rubber duck derbies. Sonya listed the data in a frequency table. Then she made a picture graph to display the data.

Rubber Duck Derbies, 2006				
Location Congaree River, SC Lake Lanier, GA St. Louis Riverfront, MO Meinig Memorial Park,				Meinig Memorial Park, OR
Number of Rubber Ducks	5,000	13,000	1 <i>5,</i> 000	1,000

A <mark>picture graph</mark> uses pictures or symbols to represent data. Each picture represents a certain amount in the data.



Explain It

1. Which is easier to interpret, a picture graph or a frequency table? Explain.

Step 1

Step 2

List the survey answers along one axis.

Choose an interval, the difference between adjoining numbers on an axis. Label both axes. Along the other axis mark the scale, the series of numbers at equal distances. Begin with 0 and include the least and greatest numbers in the survey results.

Graph the data by drawing bars of the correct length or height.

Step 4

Step 3

Title the graph.

Interpret the Graph

Most students play sports. The fewest number of students do chores.



Guided Practice*

Do you know HOW?

In **1** and **2**, decide if a bar graph or picture graph would better present the data.

- 1. The number of cats, dogs, and pet birds in a neighborhood
- 2. The number of cattle on three ranches

Do you UNDERSTAND?

- **3.** Could the data in the example of the bar graph above be presented in a picture graph? Explain.
- **4.** How are bar graphs and picture graphs similar? How are they different?

Independent Practice

In **5** through **8**, answer the questions about the picture graph shown to the right.

- 5. How many people are represented by each picture?
- **6.** What is the difference in populations between the second most populated city and the least populated city?
- **7.** About how many people live in the two most populated cities?
- **8.** Can this data be presented in a bar graph? Explain.



www.pearsonsuccessnet.com

Problem Solving

In **9** through **12**, use the bar graph.

- 9. What interval was used for the scale?
- **10.** About how many more eagle pairs were there in 1994 than in 1990?
- **11. Writing to Explain** Based on the graph, do you think the number of pairs of eagles will increase or decrease? Explain.
- **12.** Between which 2 years did the number of pairs of eagles increase the most?
- In **13** through **15**, use the frequency table.
- **13.** If you were to draw a bar graph for this frequency table, what scale would you use?
- **14.** How many more U.S. residents visited France than Italy in 2004?
- **15. Writing to Explain** Why do you think more residents went to Mexico and Canada than the other destinations?
- 16. Think About the Process Julio bought 3 dozen eggs. He had 13 eggs left after making egg salad for the picnic. Which shows how to find how many eggs Julio used?

Α	(13 – 12) × 3	C (13 × 12) − 3
В	(12 – 3) – 13	D (3 × 12) – 13

18. Point *A* represents which mixed number on this number line?





Top 5 Destinations of U.S. Residents, 2004		
Destination	Number of Travelers	
Mexico	19,360,000	
Canada	15,056,000	
United Kingdom	3,692,000	
France	2,407,000	
Italy	1,915,000	

- 17. Think About the Process A school has 12 soccer teams with 10 students on each team. The school wants to have only 8 soccer teams. Which shows how to find the number of students that would be on each team if there were only 8 teams?
 - A Multiply 10 by 8.
 - **B** Divide 120 by 8.
 - **C** Divide 8 by 120.
 - D Multiply 12 by 8.

422

1. ⁻ 5 - ⁺ 6	2. ⁺ 7 - ⁺ 3	3. ⁻ 3 - ⁺ 4
4 +8 - +9	5 ⁻ 10 - ⁺ 2	6 ⁻ 1 - ⁺ 1

Find each product. Simplify if possible.

7. $\frac{1}{2} \times \frac{6}{10}$	8. $\frac{2}{5} \times \frac{1}{5}$	9. $\frac{1}{6} \times \frac{3}{4}$	10. $8 \times \frac{5}{8}$	
11. $\frac{1}{4} \times \frac{1}{5}$	12. $\frac{3}{8} \times \frac{1}{4}$	13. $\frac{9}{10} \times \frac{1}{10}$	14. $\frac{3}{4} \times \frac{1}{3}$	15. $\frac{5}{6} \times \frac{1}{10}$
Find each quo	tient. Simplify if po	ossible.		

16. $3\frac{2}{3} \div 1\frac{5}{6}$ **17.** $1\frac{1}{2} \div 1\frac{1}{3}$ **18.** $6 \div \frac{7}{8}$ **19.** $1\frac{5}{12} \div 1\frac{5}{12}$

Error Search Find each answer that is not correct. Write it correctly and explain the error.

20.	5	21. 28,403	22. 105 R3	23. 37.91	24. 50,000
-	$-3\frac{3}{8}$	× 3	5)528	+ 14.23	× 40
-	-3	85,239		23.68	200,000
	$2\frac{2}{8}$				

Number Sense

Estimating and Reasoning Write whether each statement is true or false. Explain your reasoning.

- **25.** If a < 0 and b > 0, then a b is negative.
- **26.** The product of 25 and 2,002 is 50 less than 50,000.
- **27.** The expression (64 \div 8) + 2 × (25 \div 5) is 50.
- **28.** The sum of 22,256 and 43,008 is less than 65,000.
- **29.** The product of $6\frac{3}{5}$ and $7\frac{1}{9}$ is greater than 42.
- **30.** The quotient of $13.8 \div 2.1$ is greater than 4 and less than 7.

Practice



SDAP 1.2 Organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.

Histograms

How do you make and interpret a histogram?

A radio station recorded the ages of 25 callers in a phone survey.

This data can be shown by a histogram, a bar graph that groups data into equal intervals shown on the horizontal axis. There is no space between the bars.

Make a histogram to show the frequency of data in each age interval.

	Age	Frequency
Vata	0–19	6
	20–39	12
	40–59	5
	60–79	2

Guided Practice*

Do you know HOW?

1. The table shows the number of minutes 25 students spent on homework each night. How would the lengths of the bars compare if you made a histogram to show the data?

ata	Amount in Minutes	Frequency
9	0–29	5
	30–59	10
	60–89	5
	90–119	5

Do you UNDERSTAND?

- According to the histogram in Exercise 1, what fraction of the students surveyed spent 30–59 minutes on homework each night?
- 3. In the example above, how can you tell that $\frac{1}{5}$ of the people surveyed were in the 40–59 age group ?

Independent Practice

4. The table shows the results of a class survey about the amount of time students spend on their cell phones each day. Copy and complete the histogram shown at the right.

ata	Amount in Minutes	Frequency
6	0–9	6
	10–19	8
	20–29	10
	30–39	6







Problem Solving

One class took a survey of the amount of money they spent on CDs over 3 months and made a histogram of the results. The histogram is shown at the right.

- 5. a How many students were surveyed?
 - b What fraction of students spent between 0 and \$9.99 on CDs?
 - **c** In which range of money spent did twice as many students buy CDs than in the \$20-\$29.99 range?

- 6. Reasoning Selma says that a histogram shows that 4 times as many people in the 21–25 age group answered the survey than in the 36–40 age group. How does she know this from looking at the histogram?
- 8. On a class trip, Harry spent \$28. Nate spent \$6 less than Harry. Which expression could you use to find how much both boys spent?
 - **A** 28 + 28 + 6 **C** 28 (28 + 6)
 - **B** 28 + (28 6) **D** 28 28 6

- 7. On a coordinate grid, Sue drew a path starting at (6, 2). She moved 2 spaces to the right and 3 units up. What is the ordered pair for the point where she stopped?
- **9.** At 6 A.M., the temperature was -5° F. By noon, the temperature had increased by 12°F. What was the temperature at noon?

A	17°F	C	⁻7°F
B	7°F	D	−12°F





SDAP 1.2 Organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.

Circle Graphs

How can you use fractions and percents to label a circle graph?

A circle graph shows how all (100%) of a set of data has been divided into parts. Each part is shown by a wedge (sector) of the circle. What fraction and percent represents the part of each circle that is shaded?

Another Example How can you make a circle graph to display data?

Sixty students were asked to name their favorite sport. How could you use a circle graph to show the results?

Step 1

Use fractions to find what part of the circle should represent each category. Baseball was chosen by 30 out of 60, and $\frac{30}{60}$ is $\frac{1}{2}$ or 50%. The wedge for baseball is half the circle.

Football was chosen by 15 out of 60, and $\frac{15}{60}$ is $\frac{1}{4}$ or 25%. The wedge for football is $\frac{1}{4}$ of the circle.

Soccer was chosen by 6 out of 60, and $\frac{6}{60}$ is $\frac{1}{10}$ or 10%. The remaining part represents students who chose "other". It is $\frac{9}{60}$ or $\frac{3}{20}$ or 15% of the circle.

Step 2

Draw a circle with sectors for $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{10}$. The remaining sector represents $\frac{3}{20}$. Label the sectors to show what each part of the circle represents. Often circle graphs are labeled with a percent.

Explain It

- If 20 out of 60 students in the survey had chosen baseball, what fraction would represent the part of the circle that should be shaded to show the part of the students who chose baseball? What percent is equivalent to the fraction?
- 2. In the circles at the top of the next page, how many of the 12.5% wedges would it take to equal a $\frac{1}{4}$ wedge?
- **3.** In the circle at the right, how many wedges should be shaded to show 40%?

Favorite SportBaseball30Football15Soccer6Other9

Favorite Sport







Guided Practice*

Do you know HOW?

Use the circle below to answer 1 and 2.

- If 4 parts of the circle were shaded, what fraction and percent would be represented?
- 2. If only 1 part were shaded, what fraction and percent would be represented?

Do you UNDERSTAND?

- **3.** In a survey, 20 people out of 80 chose apple. If you made a circle graph, which sector shown in the example above would represent apple?
- **4.** Use the data below. Copy and label the graph. Label each sector with the correct color.

Independent Practice

A restaurant offers four main course choices on their dinner menu. One evening the following choices were chosen by 20 customers: chicken, 10; beef, 5; turkey, 3; ham 2.

- 5. Copy and complete the table at the right.
- **6.** Copy the circle graph at the right and label each sector with the correct main course.

Problem Solving

In two different surveys, students were asked to name their favorite type of movie. The results are shown in the table at the right.

- 7. In which survey did $\frac{1}{5}$ of the students pick comedy?
- **8.** In which survey did 50% of the students choose animated movies?
- Copy each circle graph and label it with the type of movie and the percent. You can remove the dashed lines in your final graph.
- **10.** Number Sense A circle graph is divided into three sections. One section equals 50%. The other two sections are equal in size. What percent of the circle does each of the other two sections represent?
- 12. Think About the Process Sonya spent \$18 for a book and \$22 for a DVD. She paid \$2.40 in tax and received \$7.60 in change. Which expression shows how to find the amount of money Sonya gave the clerk?
 - **A** 18 + 22 + 2.40
 - **B** 18 + 22 7.60
 - $\textbf{C} \quad 18+22+2.40-7.60$
 - **D** 18 + 22 + 2.40 + 7.60

- **11.** A triangle has a height of 16 mm and a base of 6.4 mm. What is the area of the triangle?
 - A 22.4 mm²
 C 51.2 mm²
 B 44.8 mm²
 D 102.4 mm²
- **13.** Renee mixed $\frac{3}{4}$ cup of lime juice, $\frac{7}{8}$ cup of water, and $\frac{1}{2}$ cup of ice to make a limeade. Which is a reasonable total for the amount she mixed?
 - A Less than 1 cup
 - **B** Between $1\frac{1}{2}$ cups and $2\frac{1}{2}$ cups
 - **C** More than $2\frac{1}{2}$ cups
 - D Less than 2 cups
- **14.** A survey asked 200 people to name their favorite type of fruit. The results were as follows: apple, 100; banana, 50; orange, 25; other, 25. Which graph best represents the data?

Mixed Problem Solving

Objects on Jupiter weigh about two and a half times as much as on Earth.

1. Complete the table below and then graph the values on the coordinate grid.

Earth weight	50	55				75
Jupiter weight (approx.)			150	163	175	

- 2. If a dog weighs 75 pounds on Earth, about how much would it weigh on Jupiter?
- **3.** If Tyler weighs 120 pounds on Jupiter, about how much would he weigh on Earth?
- **4.** Complete this table using the graph that shows the Big Dipper.

Point	Ordered Pair
A	
В	
С	
D	
Е	
F	
G	

- 5. If you were to move this drawing of the Big Dipper 3 units to the right on this grid, what would be the ordered pair for point *D*?
- **6. Strategy Focus** Examine the ordered pairs for Points *C* and *E*. How do you know that a vertical line can connect those two points? Explain how you decided.

MR 2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. Also **SDAP 1.2**.

Problem Solving

Make a Graph

Data for a company's sales of mountain bicycles and skateboards are shown in the table. Write two statements that compare the sales of bicycles and skateboards.

Make a line graph for each set of data to help you analyze the data.

Number of Number of Year **Bicycle Sales** : **Skateboard Sales** 2003 800 200 2004 900 400 2005 : 1,000 800 2006 1,000 999 1,100 2007 1,100

nende-On

grid paper

Guided Practice*

Do you know HOW?

 In a survey, students were asked to name their favorite pet. Copy the circle below to make a circle graph to show the data.

Dog	Cat	Bird	Other	
12	6	4	2	

2. Make a bar graph to show the data in Exercise 1.

Do you UNDERSTAND?

- **3.** In the example above, if the trend continues, what can you say about the sales of both bicycles and skateboards in 2008?
- **4. Write a Problem** Write a real-world problem that can be solved by making a graph.

Independent Practice

5. Mr. Lauer surveyed his students to find out what kind of field trip they preferred. Make a bar graph to show the data. Which field trip is most popular?

	Field Trip	Number of Votes
Date	Zoo	12
4	Aquarium	9
	Musical Play	5
	Mystery Play	4

6. Would a line graph be an appropriate graph in Exercise 5? Why or why not?

- What do I know?
- What am I asked to find?
- What diagram can l use to help understand the problem?
- Can I use addition, subtraction, multiplication, or division?
- Is all of my work correct?
- Did I answer the right question?
- Is my answer reasonable?

A survey of 16 people recorded the number of books people read in a month. Joe made a histogram and Jean made a circle graph to show the results.

Number of people	8	6	2
Number of books	0–2	3–5	6–8

- 7. Which graph makes it easy to see that $\frac{1}{2}$ of the people read 2 or less books in the month? Which graph makes it easy to tell the number of people in each category?
- 8. Writing to Explain Can you tell from the histogram how many people read 4 books?
- **9.** The data about bicycle sales and skateboard sales at the top of the page could also be shown by a double-bar graph. Part of thengraph is shown at the right. Copy and complete the graph.
- 10. A café sells turkey, roast beef, ham, or cheese sandwiches; milk, water, or juice; and yogurt or fruit. How many different meals are possible for a person who wants a sandwich, drink, and dessert?

www.pearsonsuccessnet.com

SDAP 1.1 Know the concepts of mean, median, and mode; compute and compare simple examples to show that they may differ.

Mean

How can data be described by a single number?

How can Carla find the average final score of five bowlers?

The mean, or average, <u>is the sum</u> of all the numbers in a set of data divided by the number of numbers in the set.

	-	_
9	9 10	
86 72	95 63	95
80 4 2	87 70-	87
77 ⁵¹	84 4 3 –	84
74 ²⁴	81 5 2 -	81
75 3 3	83 ⁶²	83

Guided Practice*

Do you know HOW?

In **1** through **7**, find the mean for each set of data.

- **1.** 5, 4, 4, 9, 8 **2.** 19, 55, 34, 16
- **3.** 101, 105, 103 **4.** 8, 2, 11, 6, 8
- **5.** 85, 70, 84, 91, 88, 92
- **6.** 205, 204, 398, 405, 894, 102
- **7.** 28, 32, 36, 40, 42, 57, 58, 59

Do you UNDERSTAND?

- **8.** Another team had 6 bowlers. Would the mean automatically decrease as the number of bowlers increases?
- **9.** In the example above, how could the mean be raised to 90?
- **10. Writing to Explain** Dave said that the mean of 1, 2, 3, 4, and 5 is 8. How do you know this is incorrect without finding the mean?

Independent Practice

In **11** through **22**, find the mean for each set of data.

11. 2, 5, 4, 5	12. 5, 4, 6, 9, 11	13. 6, 17, 12, 11, 4, 6, 7
14. 89, 98, 101	15. 17, 30, 45, 46, 27	16. 13, 16, 19, 21, 26
17. 35, 45, 70	18. 40, 41, 54, 55, 66, 79, 43	19. 164, 198, 301
20. 7.6, 6.2, 6.0, 7.8, 7.4	21. 11, 8.3, 9.0, 3.7	22. 129, 8,002, 1,003, 866

Problem Solving

Meredith recorded her score for each game of miniature golf she played. Use her scorecard for **23** through **25**.

- 23. What was Meredith's mean golf score?
- **24.** If Meredith had scored a 50 for the eighth game, how much would her mean score change?

Miniature Golf Scores								
Game	1	2	3	4	5	6	7	8
Score	52	56	49	51	54	52	60	58

- **25.** In miniature golf, the lower the score is, the better the game. Meredith wants to find the mean golf score of her four best games. What is this mean score?
- 27. Scientists have recorded the lengths of different species of hammerhead sharks. The lengths that have been recorded are 20 ft, 14 ft, 11 ft, and 7 ft. What is the mean length of the hammerhead shark?

A 12.5 ft C	17 ft
---------------------------	-------

- **B** 13 ft **D** 52 ft
- **29. Reasoning** A data set consisting of 3 numbers has a mean of 24. If two of the numbers are 23 and 25, what is the third number?

- **26. Geometry** Which can be modeled by light beaming from a lighthouse?
 - A Point C Ray
 - B Plane D Segment
- **28. Estimation** What is the approximate mean height of the 7 tallest peaks in Texas listed below?

	Peaks in Texas	Height in Feet
Dat a	Guadalupe Peak	8,749
	Bush Mountain	8,631
Shumard Peak		8,615
	Bartlett Peak	8,508
	Mount Livermore	8,378
	Hunter Peak	8,368
	El Capitan	8,085

SDAP 1.1 Know the concepts of mean, median, and mode; compute and compare simple examples to show that they may differ.

Median, Mode, and Range

How can data be described by one number?

Trey listed, in order, the playing times for the best-selling CD of each music type.

How can he describe the data with one number?

	CD Playing Times			
Vala	Minutes	Music Type		
	59	Popular		
	61	Country		
	63	Blues		
	63	Soundtrack		
	64	Gospel		
	67	Jazz		
	72	Classical		

Guided Practice*

Do you know HOW?

In **1** through **3**, identify the median, mode, and range for each set of data.

- **1.** 5, 7, 5, 4, 6, 3, 5
- **2.** 21, 21, 23, 32, 43
- **3.** 13, 14, 14, 16, 17, 19

For an even number of values, the median is the number halfway between the two middle values.

Do you UNDERSTAND?

- **4.** What operation is used to find the range?
- 5. In the example at the top, how would the median and mode change if the playing time for the Blues CD changed to 61 minutes?
- **6.** What would the range of playing times be if the 72–minute CD was removed from the list?

Independent Practice

In 7 through 9, use the table at the right.

- 7. What are the median, mode, and range for the data?
- **8.** What would happen to the range if the temperature were 82°F on Monday?
- **9.** If the data for Friday were removed from the table, what would the median, mode, and range be?

5-day W	5-day Weather Forecast				
Day	Temperature				
Monday	80°F				
Tuesday	80°F				
Wednesday	82°F				
Thursday	84°F				
Friday	78°F				

Find the median.

List the data from least to greatest. 59, 61, 63, <mark>63</mark>, 64, 67, 72

Identify the <mark>median</mark>, or the middle data value in an odd numbered, ordered set of data.

The median of the number of minutes of playing time is 63.

Find the mode.

59, 61, 63, 63, 64, 67, 72 Identify the mode, or <u>the data value</u> that occurs most often in the data <u>set</u>.

The mode of the number of minutes of playing time is 63.

Find the range.

59, 61, 63, 63, 64, 67, 72

Identify the range, or the difference between the greatest and least values.

72 - 59 = 13

The range of the number of minutes of playing time is 13.

Problem Solving

10. Ricardo kept a record of the 7 hottest days of the summer. Use the list below to find the median, mode, and range of the temperatures.

98°F 102°F 100°F 99°F 103°F 98°F 101°F

12. Reasoning For each statistical measure (mean, median, mode, and range) tell whether that number is always, sometimes, or never one of the numbers in the data set.

For **14** through **17**, use the table.

- **14.** What was the median number of visitors to the Statue of Liberty from May through September in 2005?
- **15.** What is the range of the data?
- **16.** How many months had over 500,000 visitors?
- **17. Writing to Explain** Why do you suppose there had been many fewer visitors in September, than in July or August?

- **11. Writing to Explain** How can you tell the difference between the net for a triangular prism and the net for a triangular pyramid?
- **13.** Think About the Process One side of a rectangular garden is 13 feet and the other side is 3 feet. Which expression shows how to find the perimeter?

A $(2 \times 13) + (2 \times 3)$ **C** $2 \times 13 \times 3$ **B** 13×3 **D** 3 + 13

Visitors to the Statue of Liberty				
2005	Visitors			
Мау	430,235			
June	492,078			
July	589,166			
August	542,292			
September	367,441			

SDAP 3.1, Grade 6 Present all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome. Also MR 3.3

Outcomes

How can tree diagrams help you list possible outcomes?

How many outcomes are possible when spinning a spinner once and then tossing a coin twice? Use a tree diagram to list all possible outcomes.

A tree diagram is a diagram used to organize outcomes of an experiment.

Guided Practice*

Do you know HOW?

In 1 through 3, list the possible outcomes.

- 1. Tossing a number cube
- 2. Spinning a spinner divided into white, blue, black, and purple
- **3.** Tossing an even number on a number cube

Do you UNDERSTAND?

- **4. Writing to Explain** In the example above, how would your tree diagram change if the spinner had 4 colors? How many possible outcomes would there be?
- **5.** Write a multiplication equation to find the possible outcomes of tossing a number cube and spinning a spinner with 4 different colors.

Independent Practice

6. Two spinners are spun. Copy and complete the tree diagram to show the possible outcomes.

Josh and Susan are running for class president. Mark, Maria, Lee, and Eva are running for vice-president. How many possible outcomes are there for electing a president and a vice president?

Problem Solving

- **8.** On four tests, Justin scored 90, 85, 90, and 95. What is the mean score of the four tests?
- **10.** John, Andy, and Miguel run in the first race. Sharon, Marie, and Mona run in the second race. How many different outcomes are possible for the winning pairs?
 - **A** 3 **B** 6 **C** 9 **D** 12
- **12. Writing to Explain** How does a tree diagram make it easier to tell which outcome occurs most often?
- 14. Think About the Process Jennifer's paycheck was \$314.79. She used \$205.75 of that money to pay bills. Then she spent \$58 on groceries. Which expression shows how to find how much money Jennifer has left?
 - **A** \$314.79 + \$205.75 \$58
 - **B** \$314.79 \$205.75 \$58
 - **C** \$314.79 \$205.75 + \$58
 - **D** \$314.79 + \$205.75 + \$58

- 9. Steve's dog is 10 lb heavier than Marsha's dog. Together, the dogs weigh 42 lb. How much does each dog weigh?
- **11.** Surfers in California often have a variety of wetsuits and surfboards. If a surfer has 3 wetsuits and 6 surfboards, how many different combinations of a wetsuit and a surfboard could the surfer have?

- 13. Lara's ice skating lesson started at 11:15 A.M. and ended at 12:50 P.M. How long did the lesson last?
- **15.** How many different outfits consisting of one pair of jeans, one T-shirt, and one jacket can you make if you have three pairs of jeans, four T-shirts, and two jackets to choose from?
- **16.** Algebra Find the value of *n*, if $n \times 400 = 28,000$.

SDAP 1.3 Use fractions compare data sets of different sizes.

Writing Probability as a Fraction

What is the probability of an event?

Reuben writes each letter of his name on a separate piece of paper and puts them in a bag. He chooses one piece of paper from the bag without looking.

The probability of an event is a number that describes the chance the event will occur.

Another Example What is the probability of two events happening together?

Eva puts the letters of her name into a bag and chooses a letter out of the bag without looking. She puts the letter back into the bag and chooses again without looking. What is the probability that Eva chooses an A both times?

Draw a tree diagram.

First Letter	Second Letter	Outcome
	E	—— E, E
E	V	—— E, V
	A —	— E, A
	_ E	—— V, E
V <<	V	— V, V
	A —	— V, A
	_ E	—— A, E
\mathbf{A}	V	— A, V
	∕_ _A	— A, A

Find the probability.

There are a total of 9 possible outcomes when the two letters are chosen and the first is replaced. One of the outcomes is favorable because only one of the outcomes has an A both times.

E

E

The probability of any event ranges from 0 to 1.

The probability that Eva chooses an A both times is $\frac{1}{0}$.

Explain It

- 1. Use the tree diagram above to find the probability of choosing the same letter twice. Hint: Look for pairs of outcomes with the same letter.
- 2. If an event has a probability of $\frac{3}{4}$, is it less likely, more likely, equally likely, impossible, or certain to occur?

number of favorable outcomes total number of possible outcomes

What is the probability that Reuben will choose the letter B?

There is 1 favorable outcome out of 6 possible outcomes, R, E, U, B, E, or N. The outcomes are equally likely (have the same chance of occurring). The probability of choosing the letter B can be written as *P*(B).

$$P(\mathsf{B}) = \frac{1}{6}$$

The probability that Reuben chooses a B out of the bag is $\frac{1}{6}$.

What is the probability that Reuben will choose the letter E?

There are 2 favorable outcomes out of 6 possible outcomes (since E appears twice).

 $P(E) = \frac{2}{6} = \frac{1}{3}$

The probability that Reuben chooses an E out of the bag is $\frac{1}{3}$ or $P(E) = \frac{1}{3}$.

Guided Practice*

Do you know HOW?

For **1** through **4**, use the spinner shown.

- 1. Find P(blue).
- 2. Find *P*(yellow).
- **3.** Find *P*(red).
- 4. Find P(green).

Do you UNDERSTAND?

- **5. Writing to Explain** In the example above, is it likely, unlikely, impossible, or certain that Reuben draws a B?
- **6.** What is the probability that Reuben will NOT draw an R?

Independent Practice

7. Write each letter of your first name on a separate small sheet of paper. Put each piece into a box. Do an experiment where you pick one letter and replace it each time. Do this 10 times. Record the number of times you pick each letter, and then write the probability as a fraction.

For **8** through **12**, suppose you toss a quarter and a penny.

- 8. Make a tree diagram to show the possible outcomes.
- 9. Find P(one head).
- **10.** Find *P*(two heads).
- 11. Find P(quarter heads, penny tails)
- **12.** Find *P*(no heads)

*For another example, see Set H on page 449.

13. When a number cube is tossed, there are 6 possible outcomes (1, 2, 3, 4, 5, or 6). If the cube is tossed twice and the outcomes are added, the possible sums are from 2 through 12.

Copy the table and give the probability of each sum.

Sum	2	3	4	5	6	7	8	9	10	11	12
Number of Occurrences	1	2	3	4	5	6	5	4	3	2	1
Probability											

- 14. Which sum (or sums) has the greatest probability of occurring?
- 15. Which sum (or sums) has the least probability of occurring?

Problem Solving

- **16. Geometry** Kendra tosses a colored cube. Half of the sides of the cube are red, $\frac{1}{3}$ of the sides are blue, and one side is green. What is the probability that the cube will land on a color other than red when tossed?
- 18. Which About the Process Jorgé put colored cards into a bag. Two of the cards were green, three were red, one was orange, two were blue, and two were purple. Jorgé wants to find the probability that he will pull an orange card from the bag. What step does Jorgé take to determine the number of possible outcomes?
 - **A** Count the number of orange cards.
 - **B** Count the number of different colored cards.
 - **C** Count the total number of cards in the bag.
 - **D** Count the number of cards that are not orange.

- 17. Mrs. Pierre bought 150 pencils to give to her students. She has three classes with 27, 25, and 23 students each. She wants every student to get the same number of pencils. How many pencils should she give to each student?
- **19.** Carlita buys 3 beanbag throws for \$1.00. What is the probability she will toss one beanbag through a hole in the top row of this game? Assume Carlita always throws a beanbag into a hole.

20. How many parts of each color should there be to make sure that it is equally

likely this spinner will land on each of 3 different colors?

Which Equation is True?

Remember that an equation is a number sentence that uses an equal sign to show that two expressions are equal. Both of the following are equations.

x + 25 = 100

The first equation is true. You don't know if an algebraic equation is true or false until you replace the variable with a number.

If x = 75, then x + 25 = 100 is a true equation.

Example: If m = 25, which equation is true?

2 + m = 30 m - 20 = 5

In the first equation, if you replace m with 25, the result is 2 + 25 = 30. This equation is false.

In the second equation if you replace m with 25, the result is 25 - 20 = 5. This equation is true.

Decide which equation is true for each replacement of the variable.

- **1.** If x = 20, which equation is true?
 - 3 + x = 23 or $\frac{x}{5} = 2$
- **3.** If x = 50, which equation is true? x + 50 = 75 or $\frac{x}{10} = 5$
- **5.** If z = 200, which equation is true? 200 - x = 200 or x + 100 = 300
- **7.** If x = 0, which equation is true?
 - 50x = 50 x + 7 = 7

- **2.** If y = 100, which equation is true? y - 80 = 180 or $y \div 2 = 50$
- **4.** If x = 80, which equation is true? $40x = 3,200 \text{ or } \frac{x}{4} = 2$
- **6.** If x = 70, which equation is true? 6x = 420 or x + 7 = 63
- **8.** If x = 1, which equation is true?

40x = 40 or 40 + x = 40

9. If a teacher has 32 students and divides them into teams of 4, which equation could be used to find how many students will be on each team? Let *x* represent the number of students on each team.

$$\frac{32}{4} = x$$
 $4 + x = 32$ $\frac{32}{x} = 4$ $4 \times 32 = x$

MR 1.2 Determine when and how to break a problem into simpler parts. Also NS 1.0, MR 2.2.

Problem Solving

Solve a Simpler Problem

Owen needs to read 2 books from a list of 6 books. How many different combinations of books are possible?

Follow these steps to solve a simpler problem.

- 1. Break apart or change the problem into one that is simpler.
- 2. Solve the simpler problem.
- 3. Use the answers to the simpler problem to solve the original problem.

Guided Practice*

Do you know HOW?

- 1. Draw a picture to show the number of combinations of pairs for 5 books.
- **2.** Think of extending the table at the top to find the number of pairs with 7 books. What number would you add to 15 to find the number of pairs? How many pairs would there be for 7 books?

Do you UNDERSTAND?

3. Writing to Explain Is it easier to use the table or to draw a picture as the number of books increases?

books from

reading list

4. Write a Problem Write a real-world problem that can be solved by solving a simpler problem.

Independent Practice

Solve each problem.

- 5. Continue the pattern in the book problem above. How many pairs of books would there be for 8 books? 9 books? 10 books?
- 6. Find the number of degrees in a hexagon. HINT: Divide the hexagon into triangles.
 - **a** How many triangles are formed?
 - **b** How many degrees are in each triangle?
 - c What is the total number of degrees in the hexagon?
- 7. Using the same strategy in Problem 6, what is the total number of degrees in a pentagon? An octagon?

8. Juanita tossed 3 number cubes, and these digits came up.

If each of the numbers 2, 1 and 4 is used only once to form a 3-digit number, which shows all the possible 3-digit numbers?

- **A** 214, 421, 142
- **B** 214, 124, 412, 421
- **C** 214, 241, 142, 412, 124
- **D** 214, 241, 142, 124, 412, 421
- 10. George was choosing his clothes for the next day. In his closet he had 2 ties, 5 shirts, 3 trousers, and 2 belts. How many different ways could George choose a tie, shirt, trousers, and belt?
- **12.** After seeing a movie, 2 friends stopped for frozen yogurt. Three flavors were available in small, medium, and large sizes. How many different combinations of flavors and sizes are possible?

9. Jill has 3 colored vases to arrange on a shelf.

Let B stand for blue, O for orange, and G for green. Which list shows all the possible arrangements of the vases?

- **A** BOG, OBG, GOB
- **B** BOG, OBG, GOB, GBO
- C BOG, OBG, GOB, GBO, OGB
- **D** BOG, OBG, GOB, GBO, OGB, BGO
- 11. The McMillan family wanted to buy a new vehicle. They could choose a van or a car; a black, silver, or white exterior; and a tan or black interior. How many different vehicles can they buy?
- **13. Algebra** Draw a picture and write an equation to solve.

Niko had $17\frac{1}{3}$ ft of fencing. He uses $5\frac{2}{3}$ ft to finish a job. How many feet of fencing does Niko have now? Let f = feet of fencing left.

Test Prep

 The histogram shows the results from a survey asking people how many plays they have seen in the last year. How many people have seen more than 3 plays? (19-2)

- **A** 26
- **B** 15
- **C** 11
- **D** 8
- 2. The music teacher is choosing two students out of 10 to sing a duet in the musical. How many different pairs of students must try out if she wants to try every possible pair? (19-9)
 - **A** 50
 - **B** 45
 - **C** 20
 - **D** 12

3. James is drawing the picture graph below to show the number of license plates from each state he saw on a recent trip. He saw 15 Arizona license plates. Which picture should he draw for Arizona? (19-1)

License Plates James Saw				
State	Number of License Plates			
Nevada	<u>Şoog-nini) Şoog-nini) Şoog-nini) Şoog-nini) Şoog-nini)</u>			
Oklahoma	<u>\$000-nnn } 5001</u>			
New Mexico	<u>\$000-HNN</u> <u>\$000-HNN</u>			
Arizona				
Arkansas	\$000-HNN			
Other States	<u>\$000-HINN</u> <u>\$001</u>			
E 5 license plates				
	NNN <u>7000-NNN</u> <u>7001</u>			
B [300-11111] [300-11111] [300-11111] [300-11111]				

- D 2000-NNN 2000-NNN 2000-NNN
- **4.** The shoe sizes of the starting players on the girls' basketball team are listed below. What is the median of these numbers? (19-6)
 - 7, 6, 5, 6, 8
 - **A** 5
 - **B** 6
 - **C** 7
 - **D** 8

5. According to the circle graph below, about what fraction of the t-shirts sold were x-large? (19-3)

6. The table shows the heights of a group of friends. What is the mean height of the group in inches? (19-5)

	Name	Height (in inches)
Data	Amy	58
	Rudy	62
	Jessie	55
	Pablo	61
	Jared	64

- **A** 61
- **B** 60
- **C** 59
- **D** 58

- 7. A college student plans to take biology, algebra, and literature in the morning. Each letter represents the first letter of each class. Which list below shows all the different orders that these three classes can be taken? (19-7)
 - A BAL, BAB, LAB
 - B BAL, LAB, ABL, LBA
 - C BAL, BLA, ALB, LAB, ABL
 - D BAL, BLA, ALB, ABL, LAB, LBA
- 8. The cards shown are placed in a bag and one is drawn without looking.
 What is the probability Bobby's name is drawn? (19-8)

- **9.** An airline company is making a graph to display the number of each type of ticket they have available on a flight including first class, business class and economy. Which type of graph would be best to display the data? (19-4)
 - A Bar or picture graph
 - **B** Line plot
 - C Circle graph
 - **D** Line graph

Set A, pages 420-422

Students were asked to name their favorite seasons. Make a bar graph to show the results.

	Favorite Sea	son
Date	Summer	8
	Autumn	6
	Winter	4
	Spring	6

Remember that a bar graph or a picture graph can be used to compare data that shows how many or how much. In a picture graph, each symbol represents a certain amount in the data.

1. Students were asked to name their favorite flower. Make a bar graph to show the results.

	Favorite Flower						
	Rose	10					
	Lily	3					
	Carnation	3					
	Tulip	6					
Į	Daffodil	8					

Set B, pages 424-425

David made a histogram to show how many books his class had purchased in the last year.

Using the histogram, find the number of students who purchased 3–5 books. The bar with the 3–5 interval has a height of 8. The number of students is 8.

Remember that histograms are a type of bar graph that has no space between the bars and the bars show equal intervals.

Use the histogram at the left to answer the following questions.

- 1. How many students were surveyed?
- 2. How many students had bought more than 5 books?
- **3.** What fraction of the students bought 9, 10, or 11 books?

Reteaching

Comedy

4

Other <u>3</u> 20

Sports

 $\frac{1}{2}$

Set C, pages 426–428

A group of 100 students were asked to name their favorite type of television program. Make a circle graph to show the results.

	Favorite Typ	e of Televis	ion Progra	m
ata	Comedy	•	25	
9	Sports	:	50	
	Drama	•	10	
	Other	•	15	

Comedy = 25 out of $100 = \frac{1}{4}$.

Sports = 50 out of $100 = \frac{1}{2}$,

Drama = 10 out of $100 = \frac{1}{10}$.

Other
$$=\frac{15}{100}=\frac{3}{20}$$
.

Draw a circle with sectors for $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{10}$ and label those sectors with the program type. The remaining sector shows $\frac{3}{20}$ or Other. **Remember** a circle graph shows the whole amount (100%) and each sector represents a part of the whole amount.

Jill spent a total of 30 hours exercising last month: Jogging: 15 hours; Cycling: 10 hours; Swimming: 5 hours.

- 1. Copy the circle graph and label each sector with the activity and fraction.
- **2.** What fraction represents the part of the day Jill spent cycling?

Set D, pages 430-431

Favorite AnimalPenguin7Elephant6Lion3Monkey4

Make a bar graph to show the results.

Students were asked to name their favorite animal.

Remember that making a graph makes it easy to visualize data and answer questions about the data.

Monthly Snowfall					
Month	Dec.	Jan.	Feb.	Mar.	
Calories	6	12	18	4	

- **1.** Make a line graph of the data.
- 2. Which month has a snowfall 3 times as great as December?

Reteaching

Set E, pages 432–433

Find the mean of this data set: 2, 3, 8, 3, 5, 6, 1

The mean is an average. To find the mean, add the data and divide by the number of data.

 $(2 + 3 + 8 + 3 + 5 + 6 + 1) \div 7 = 28 \div 7 = 4$

The mean is 4.

Set F, pages 434-435

Find the median, mode, and range for this set of data: 10, 15, 20, 12, 10

To find the median, list the data in order from least to greatest and find the middle value.

10, 10, 12, 15, 20 **The median is 12.**

To find the mode, list the data in order from least to greatest and find the value that occurs most often.

10, 10, 12, 15, 20 **The mode is 10.**

To find the range, subtract the least value from the greatest value.

20 - 10 = 10 **The range is 10.**

Set G, pages 436–437

Judy can pick either a hamburger or a chicken sandwich and then either a coleslaw, potato salad, or a fruit salad. How many different lunch combinations can she have?

Draw a tree diagram.

There are 6 possible combinations, so Judy has a choice of 6 different lunches.

Remember that the mean is an average. You must add the data and divide by the number of addends.

5, 4, 1, 3, 1, 10
 6, 9, 5, 2, 8
 4, 2, 2, 5, 3, 3, 2

Remember that if there is an even number of data, you must add the two middle numbers when the data are ordered from least to greatest and divide by 2 to find the median.

- Find the median of this data set: 27.5 27 30 29.5
- **2.** Find the mode of this data set: 12, 6, 9, 5, 8, 12, 8, 1, 4, 12, 6
- **3.** Find the range of this data set: 87, 84, 90, 75, 100, 88

Remember that when you make a tree diagram, start with 1 item and list all the possible combinations with it. Continue with all the items.

- Make a tree diagram to show the sample space for tossing a number cube and then tossing a quarter.
- 2. Make a tree diagram to show the results of choosing among tuna, cheese, and turkey sandwiches, and milk and juice drinks.

Set H, pages 438-440

You spin the spinner once. Find *P* (landing in a green section).

 $Probability = \frac{number of favorable outcomes}{number of possible outcomes}$

So, *P* (green) $= \frac{3}{6} = \frac{1}{2}$

Set I, pages 442–443

To solve a simpler problem, follow these steps:

Break apart or change the problem into one that is simpler to solve.

Draw a picture or use objects to look for a pattern to solve the simpler problem.

Step 3

Use the answers to the simpler problem to solve the original problem.

Four people shake hands with each other once. How many handshakes are there in all?

Use A, B, C, D to represent the people.

AB, AC, AD BC, BD, CD

There are 6 handshakes.

Remember that when you express a probability as a fraction, always write the fraction in simplest form.

Use the letters in MATHEMATICS. Each letter is written on a separate piece of paper and the pieces are put in a bag.

Find the probability of choosing each letter or letters out of the bag.

- 1. P (a consonant)
- 2. P (the letter A)
- **3.** *P* (not the letter *A*)

Remember that you can draw a picture or use objects to look for a pattern in finding the relationship between the simpler problem and the original problem.

- How many different teams of 2 people can be chosen from 4 people?
- How many different teams of 3 people can be chosen from 5 people?
- **3.** What is the sum of the angles of an octagon?