

3
In 2006, 13,000 rubber ducks raced in the 2006 Rubber Duck Derby in Lake Lanier, Georgia. Other locations across the U.S. had derbies that same year. Which derby had more ducks than the one in Georgia? You will find out in Lesson 19-1.


Review What You Know!

Vocabulary
Choose the best term from the box.

- axis
- point
- ordered pair

1. $A(n)$ ? is either of two perpendicular lines in a graph.
2. $A(n)$ ? is an exact location in space.
3. A pair of numbers used to locate a point on a graph is called $a(n)$ ?

Number Lines
Use the number line to answer 4 through 6.

4. Which point is halfway between 1 and 2 ?
5. Which point is closest to 1 than to 2 ?
6. Which point is closest to 2 than to 1 ?

Writing Fractions
7. If 3 out of 6 marbles are red, what fraction, in simplest form, names the red marbles?
8. If 1 out of 5 apples is green, what fraction names the part of the apples that are NOT green?

Division Patterns
Writing to Explain Write an answer to the question.
9. How is the number of zeros in the quotient of $45,000 \div 9$ related to the number of zeros in the dividend?

Also SDAP 1．3．

## Bar Graphs and Picture Graphs

## How do you display data collected from a count or measure？

Students were surveyed about what they do after school．The results were displayed in a bar graph．
A bar graph uses rectangles（bars）to show and compare data that tells how many or how much．

## After－School Activities

| Sports | 䀘册册 |
| :---: | :---: |
| Homework | 册 IIII |
| Chores | 册I |
| Other | H册 II |

## Another Example How can you make and interpret picture graphs？

Sonya gathered data about the number of ducks in some of the 2006 rubber duck derbies．Sonya listed the data in a frequency table． Then she made a picture graph to display the data．

| Rubber Duck Derbies，2006 |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Location | Congaree River，SC | Lake Lanier，GA | St．Louis Riverfront，MO | Meinig Memorial Park，OR |
| Number of <br> Rubber Ducks | 5,000 | 13,000 | 15,000 | 1,000 |

A picture graph uses pictures or symbols to represent data． Each picture represents a certain amount in the data．

| Rubber Duck Derbies， 2006 |  |
| :--- | :--- | :--- |
| Location | Number of Rubber Ducks |
| Congaree River，SC |  |
| Lake Lanier，GA |  |
| St．Louis Riverfront，MO |  |
| Meinig Memorial Park，OR |  |
| Key：＝1，000 rubber ducks |  |

## Gxplain 18

1．Which is easier to interpret，a picture graph or a frequency table？Explain．

Step 1 List the survey answers along one axis.

## Step 2

Choose an interval, the difference between adjoining numbers on an axis. Label both axes. Along the other axis mark the scale, the series of numbers at equal distances. Begin with 0 and include the least and greatest numbers in the survey results.

Graph the data by drawing bars of the correct length or height.

Step 4 Title the graph.

## Interpret the Graph

Most students play sports. The fewest number of

## After-School Activities

 students do chores.

## Guided Practice*

## Do you know HOW?

In 1 and $\mathbf{2}$, decide if a bar graph or picture graph would better present the data.

1. The number of cats, dogs, and pet birds in a neighborhood
2. The number of cattle on three ranches

## Do you UNDERSTAND?

3. Could the data in the example of the bar graph above be presented in a picture graph? Explain.
4. How are bar graphs and picture graphs similar? How are they different?

## Independent Practice

In 5 through 8, answer the questions about the picture graph shown to the right.
5. How many people are represented by each picture?
6. What is the difference in populations between the second most populated city and the least populated city?
7. About how many people live in the two most populated cities?
8. Can this data be presented in a bar graph? Explain.

## Top 5 U.S. Cities by Population

New York


Los Angeles
Chicago
Houston
Philadelphia
Key: 酋 $^{=} 1$ million people

In 9 through 12, use the bar graph.
9. What interval was used for the scale?
10. About how many more eagle pairs were there in 1994 than in 1990?
11. Writing to Explain Based on the graph, do you think the number of pairs of eagles will increase or decrease? Explain.
12. Between which 2 years did the number of pairs of eagles increase the most?

In 13 through 15, use the frequency table.
13. If you were to draw a bar graph for this frequency table, what scale would you use?
14. How many more U.S. residents visited France than Italy in 2004?
15. Writing to Explain Why do you think more residents went to Mexico and Canada than the other destinations?
16. Think About the Process Julio bought 3 dozen eggs. He had 13 eggs left after making egg salad for the picnic. Which shows how to find how many eggs Julio used?
A $(13-12) \times 3$
C $(13 \times 12)-3$
B $(12-3)-13$
D (3 $\times 12$ ) -13
18. Point $A$ represents which mixed number on this number line?


Estimated Number of Bald Eagle Pairs in U.S.


| Top 5 Destinations of U.S. Residents, 2004 |  |
| :--- | :---: |
| Destination | Number of Travelers |
| Mexico | $19,360,000$ |
| Canada | $15,056,000$ |
| United Kingdom | $3,692,000$ |
| France | $2,407,000$ |
| Italy | $1,915,000$ |

17. Think About the Process A school has 12 soccer teams with 10 students on each team. The school wants to have only 8 soccer teams. Which shows how to find the number of students that would be on each team if there were only 8 teams?

A Multiply 10 by 8 .
B Divide 120 by 8 .
C Divide 8 by 120 .
D Multiply 12 by 8 .

Find each difference.

1. $-5-+6$
2. $+7-+3$
3. $-3-+4$
4. $+8-+9$
5. $-10-+2$
6. $-1-+1$

Find each product. Simplify if possible.

7. $\frac{1}{2} \times \frac{6}{10}$
8. $\frac{2}{5} \times \frac{1}{5}$
9. $\frac{1}{6} \times \frac{3}{4}$
10. $8 \times \frac{5}{8}$
11. $\frac{1}{4} \times \frac{1}{5}$
12. $\frac{3}{8} \times \frac{1}{4}$
13. $\frac{9}{10} \times \frac{1}{10}$
14. $\frac{3}{4} \times \frac{1}{3}$
15. $\frac{5}{6} \times \frac{1}{10}$

Find each quotient. Simplify if possible.
16. $3 \frac{2}{3} \div 1 \frac{5}{6}$
17. $1 \frac{1}{2} \div 1 \frac{1}{3}$
18. $6 \div \frac{7}{8}$
19. $1 \frac{5}{12} \div 1 \frac{5}{12}$

Error Search Find each answer that is not correct. Write it correctly and explain the error.
20. $\begin{array}{r}5 \\ -\quad 3 \frac{3}{8} \\ \hline 2 \frac{3}{8}\end{array}$
21. 28,403
22. $\frac{105}{5}{ }_{53}^{528}$
23. 37.91
$\begin{array}{r} \\ \times \quad 3 \\ \hline 85,239\end{array}$

$$
\begin{array}{r}
+\quad 14.23 \\
\hline 23.68
\end{array}
$$

24. 50,000

| $\times \quad 40$ |
| :--- |
| 200,000 |

## Number Sense

Estimating and Reasoning Write whether each statement is true or false.
Explain your reasoning.
25. If $a<0$ and $b>0$, then $a-b$ is negative.
26. The product of 25 and 2,002 is 50 less than 50,000 .
27. The expression $(64 \div 8)+2 \times(25 \div 5)$ is 50 .
28. The sum of 22,256 and 43,008 is less than 65,000 .
29. The product of $6 \frac{3}{5}$ and $7 \frac{1}{9}$ is greater than 42 .
30. The quotient of $13.8 \div 2.1$ is greater than 4 and less than 7 .

## Histograms

How do you make and interpret a histogram?
A radio station recorded the ages of 25 callers in a phone survey.
This data can be shown by a histogram, a bar graph that groups data into equal intervals shown on the horizontal axis. There is no space between the bars.
Make a histogram to show the frequency of data in each age interval.

Age | Arequency |
| :---: |
| $0-19$ |
| $20-39$ |
| $40-59$ |
| $60-79$ |

## Guided Practice*

## Do you know HOW?

1. The table shows the number of minutes 25 students spent on homework each night. How would the lengths of the bars compare if you made a histogram to show the data?


## Do you UNDERSTAND?

2. According to the histogram in Exercise 1, what fraction of the students surveyed spent 30-59 minutes on homework each night?
3. In the example above, how can you tell that $\frac{1}{5}$ of the people surveyed were in the 40-59 age group ?

## Independent Practice

4. The table shows the results of a class survey about the amount of time students spend on their cell phones each day. Copy and complete the histogram shown at the right.

| Amount in |  |
| :---: | :---: |
| Minutes | Frequency |
| $0-9$ | $\vdots$ |
| $10-19$ | 6 |
| $20-29$ | 8 |
| $30-39$ | 10 |



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Step 1
List the age intervals along the horizontal axis.

## Step 2

Along the vertical axis mark the scale. List the greatest and least numbers in the survey results. Choose an interval. Label the axes.

Step 3
Graph the data by drawing bars of the correct height. Title the graph.

## Radio Survey



Interpret the graph. Twice as many people were in the 20-39 age group as in the 0-19 age group. Most people were in the 20-39 age group.
The group with the least number was the 60-79 age group.

## Problem Solving

One class took a survey of the amount of money they spent on CDs over 3 months and made a histogram of the results. The histogram is shown at the right.
5. a How many students were surveyed?
b What fraction of students spent between 0 and $\$ 9.99$ on CDs?
c In which range of money spent did twice as many students buy CDs than in the $\$ 20-\$ 29.99$ range?

Money Spent on CDs

6. Reasoning Selma says that a histogram shows that 4 times as many people in the $21-25$ age group answered the survey than in the 36-40 age group. How does she know this from looking at the histogram?
8. On a class trip, Harry spent $\$ 28$. Nate spent \$6 less than Harry. Which expression could you use to find how much both boys spent?
A $28+28+6$
C $28-(28+6)$
B $28+(28-6)$
D 28-28-6
7. On a coordinate grid, Sue drew a path starting at ( 6,2 ). She moved 2 spaces to the right and 3 units up. What is the ordered pair for the point where she stopped?
9. At 6 A.M., the temperature was $-5^{\circ} \mathrm{F}$. By noon, the temperature had increased by $12^{\circ} \mathrm{F}$. What was the temperature at noon?
A $17^{\circ} \mathrm{F}$
C ${ }^{-7}{ }^{\circ} \mathrm{F}$
B $7^{\circ} \mathrm{F}$
D ${ }^{-1} 2^{\circ} \mathrm{F}$

SDAP 1.2 Organize and
display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.

## Circle Graphs

## How can you use fractions and percents to label a circle graph?

A circle graph shows how all (100\%) of a set of data has been divided into parts. Each part is shown by a wedge (sector) of the circle. What fraction and percent represents the part of each circle that is shaded?


## Another Example How can you make a circle graph to display data?

Sixty students were asked to name their favorite sport.
How could you use a circle graph to show the results?
Step 1
Use fractions to find what part of the circle should represent each category. Baseball was chosen by 30 out of 60 , and $\frac{30}{60}$ is $\frac{1}{2}$ or $50 \%$. The wedge for baseball is half the circle.
Football was chosen by 15 out of 60 , and $\frac{15}{60}$ is $\frac{1}{4}$ or $25 \%$.
The wedge for football is $\frac{1}{4}$ of the circle.
Soccer was chosen by 6 out of 60 , and $\frac{6}{60}$ is $\frac{1}{10}$ or $10 \%$.
The remaining part represents students who chose "other". It is $\frac{9}{60}$ or $\frac{3}{20}$ or $15 \%$ of the circle.

Favorite Sport
Baseball : 30
Football : 15
Soccer : 6
Other : 9

Favorite Sport


Step 2
Draw a circle with sectors for $\frac{1}{2}, \frac{1}{4}$, and $\frac{1}{10}$. The remaining sector represents $\frac{3}{20}$. Label the sectors to show what each part of the circle represents. Often circle graphs are labeled with a percent.

## Explainll

1. If 20 out of 60 students in the survey had chosen baseball, what fraction would represent the part of the circle that should be shaded to show the part of the students who chose baseball? What percent is equivalent to the fraction?
2. In the circles at the top of the next page, how many of the $12.5 \%$ wedges would it take to equal a $\frac{1}{4}$ wedge?
3. In the circle at the right, how many wedges should be shaded to show $40 \%$ ?


In the first circle graph, $\frac{1}{5}$ or $20 \%$ is shaded. In the second circle, $\frac{1}{4}$ or $25 \%$ is shaded.


Some other useful fractions for circle graphs are $\frac{1}{3}, \frac{1}{6}, \frac{1}{8}$, and $\frac{1}{10}$.


## Guided Practice*

## Do you know HOW?

Use the circle below to answer 1 and 2.


1. If 4 parts of the circle were shaded, what fraction and percent would be represented?
2. If only 1 part were shaded, what fraction and percent would be represented?

## Do you UNDERSTAND?

3. In a survey, 20 people out of 80 chose apple. If you made a circle graph, which sector shown in the example above would represent apple?
4. Use the data below. Copy and label the graph. Label each sector with the correct color.


## Independent Practice

A restaurant offers four main course choices on their dinner menu. One evening the following choices were chosen by 20 customers: chicken, 10; beef, 5; turkey, 3; ham 2.
5. Copy and complete the table at the right.
6. Copy the circle graph at the right and label each sector with the correct main course.

|  | Fraction | Percent |
| :--- | :--- | :--- |
| Chicken |  |  |
| Beef |  |  |
| Turkey |  |  |
| Ham |  |  |



## Problem Solving

In two different surveys, students were asked to name their favorite type of movie. The results are shown in the table at the right.
7. In which survey did $\frac{1}{5}$ of the students pick comedy?
8. In which survey did $50 \%$ of the students choose animated movies?
9. Copy each circle graph and label it with the type of movie and the percent. You can remove the dashed lines in your final graph.
10. Number Sense A circle graph is divided into three sections. One section equals $50 \%$. The other two sections are equal in size. What percent of the circle does each of the other two sections represent?
12. Think About the Process Sonya spent $\$ 18$ for a book and $\$ 22$ for a DVD. She paid $\$ 2.40$ in tax and received $\$ 7.60$ in change. Which expression shows how to find the amount of money Sonya gave the clerk?

A $18+22+2.40$
B $18+22-7.60$
C $18+22+2.40-7.60$
D $18+22+2.40+7.60$

11. A triangle has a height of 16 mm and a base of 6.4 mm . What is the area of the triangle?
A $22.4 \mathrm{~mm}^{2}$
C $51.2 \mathrm{~mm}^{2}$
B $44.8 \mathrm{~mm}^{2}$
D $102.4 \mathrm{~mm}^{2}$
13. Renee mixed $\frac{3}{4}$ cup of lime juice, $\frac{7}{8}$ cup of water, and $\frac{1}{2}$ cup of ice to make a limeade. Which is a reasonable total for the amount she mixed?

A Less than 1 cup
B Between $1 \frac{1}{2}$ cups and $2 \frac{1}{2}$ cups
C More than $2 \frac{1}{2}$ cups
D Less than 2 cups
14. A survey asked 200 people to name their favorite type of fruit. The results were as follows: apple, 100; banana, 50; orange, 25; other, 25 . Which graph best represents the data?
A

B

C

D


Objects on Jupiter weigh about two and a half times as much as on Earth.

1. Complete the table below and then graph the values on the coordinate grid.

| Earth <br> weight | 50 | 55 | $\square$ | $\square$ | $\square$ | 75 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Jupiter <br> weight <br> (approx.) | $\square$ |  | 150 | 163 | 175 | $\square$ |

2. If a dog weighs 75 pounds on Earth, about how much would it weigh on Jupiter?

3. If Tyler weighs 120 pounds on Jupiter, Pounds on Earth about how much would he weigh on Earth?
4. Complete this table using the graph that shows the Big Dipper.

| Point | Ordered Pair |
| :---: | :---: |
| $A$ |  |
| $B$ |  |
| $C$ |  |
| $D$ |  |
| $E$ |  |
| $F$ |  |
| $G$ |  |


5. If you were to move this drawing of the Big Dipper 3 units to the right on this grid, what would be the ordered pair for point $D$ ?
6. Strategy Focus Examine the ordered pairs for Points $C$ and $E$. How do you know that a vertical line can connect those two points? Explain how you decided.

Lesson
$19-4$
(1)

MR 2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. Also SDAP 1.2.

## Make a Graph

Data for a company's sales of mountain bicycles and skateboards are shown in the table. Write two statements that compare the sales of bicycles and skateboards.

Make a line graph for each set of data to help you analyze the data.

## Guided Practice*

## Do you know HOW?

1. In a survey, students were asked to name their favorite pet. Copy the circle below to make a circle graph to show the data.

| Dog | Cat | Bird | Other |
| :---: | :---: | :---: | :---: |
| 12 | 6 | 4 | 2 |


2. Make a bar graph to show the data in Exercise 1.

## Do you UNDERSTAND?

3. In the example above, if the trend continues, what can you say about the sales of both bicycles and skateboards in 2008?
4. Write a Problem Write a real-world problem that can be solved by making a graph.

## Independent Practice

5. Mr. Lauer surveyed his students to find out what kind of field trip they preferred. Make a bar graph to show the data. Which field trip is most popular?

| Field Trip | Number of Votes |  |
| :--- | :---: | :---: |
| Zoo | Num....................................... | 12 |
| Aquarium | 9 |  |
| Musical Play |  |  |
| Mystery Play | 5 |  |

6. Would a line graph be an appropriate graph in Exercise 5? Why or why not?

- What do I know?
- What am I asked to find?
- What diagram can I use to help understand the problem?
- Can I use addition, subtraction, multiplication, or division?
- Is all of my work correct?
- Did I answer the right question?
- Is my answer reasonable?

Plan and Solve
I can make a line graph for bicycle sales.

I can make a line graph for skateboard sales.

The sales for bicycles and skateboards have been increasing over the years.

The difference between the number of bicycle sales and the number of skateboard sales is becoming smaller.

A survey of 16 people recorded the number of books people read in a month. Joe made a histogram and Jean made a circle graph to show the results.

| Number of <br> people | 8 | 6 | 2 |
| :--- | :---: | :---: | :---: |
| Number of <br> books | $0-2$ | $3-5$ | $6-8$ |

7. Which graph makes it easy to see that $\frac{1}{2}$ of the people read 2 or less books in the month? Which graph makes it easy to tell the number of people in each category?

Histogram
Books Read by 16 People in 1 Month


Number of Books

## Circle graph

Books Read by 16 People in 1 month

8. Writing to Explain Can you tell from the histogram how many people read 4 books?
9. The data about bicycle sales and skateboard sales at the top of the page could also be shown by a double-bar graph. Part of thengraph is shown at the right. Copy and complete the graph.
10. A café sells turkey, roast beef, ham, or cheese sandwiches; milk, water, or juice; and yogurt or fruit. How many different meals are possible for a person who wants a sandwich, drink, and dessert?

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## Mean

How can data be described by a single number?
How can Carla find the average final score of five bowlers? The mean, or average, is the sum of all the numbers in a set of data divided by the number of numbers in the set.

| 9 | 10 | $\begin{array}{\|l\|} \hline \text { FINAL } \\ \text { SCORE } \end{array}$ |
| :---: | :---: | :---: |
| $86^{7 / 2}$ | $95^{\text {6/31 }}$ | 95 |
| $80^{142}$ | 87 7101- | 87 |
| $77{ }^{5} 511$ | $84^{4 / 31}$ | 84 |
| $74.2{ }^{4}$ | $81^{5 / 51}$ | 81 |
| 75 3/3 | 83 \|61 | 83 |

## Guided Practice*

## Do you know HOW?

In 1 through 7, find the mean for each set of data.

1. $5,4,4,9,8$
2. $19,55,34,16$
3. $101,105,103$
4. $8,2,11,6,8$
5. $85,70,84,91,88,92$
6. $205,204,398,405,894,102$
7. $28,32,36,40,42,57,58,59$

## Do you UNDERSTAND?

8. Another team had 6 bowlers. Would the mean automatically decrease as the number of bowlers increases?
9. In the example above, how could the mean be raised to 90 ?
10. Writing to Explain Dave said that the mean of $1,2,3,4$, and 5 is 8 . How do you know this is incorrect without finding the mean?

## Independent Practice

In 11 through 22, find the mean for each set of data.
11. $2,5,4,5$
12. $5,4,6,9,11$
13. $6,17,12,11,4,6,7$
14. $89,98,101$
15. $17,30,45,46,27$
16. $13,16,19,21,26$
17. $35,45,70$
18. $40,41,54,55,66,79,43$
19. $164,198,301$
20. $7.6,6.2,6.0,7.8,7.4$
21. $11,8.3,9.0,3.7$
22. $129,8,002,1,003,866$

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Add the final scores in the set of data.

## 95

87
84
81
$\begin{array}{r}+\quad 83 \\ \hline 430 \text { sum }\end{array}$

Divide the sum by the number of numbers in the set.


The average, or mean, score for the 5 bowlers is 86 .

## Problem Solving

Meredith recorded her score for each game of miniature golf she played. Use her scorecard for 23 through 25.
23. What was Meredith's mean golf score?
24. If Meredith had scored a 50 for the eighth game, how much would her mean score change?
25. In miniature golf, the lower the score is, the better the game. Meredith wants to find the mean golf score of her four best games. What is this mean score?
27. Scientists have recorded the lengths of different species of hammerhead sharks. The lengths that have been recorded are $20 \mathrm{ft}, 14 \mathrm{ft}, 11 \mathrm{ft}$, and 7 ft . What is the mean length of the hammerhead shark?
A 12.5 ft
C 17 ft
B 13 ft
D 52 ft
29. Reasoning A data set consisting of 3 numbers has a mean of 24 . If two of the numbers are 23 and 25 , what is the third number?

| Miniature Golf Scores |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Game | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Score | 52 | 56 | 49 | 51 | 54 | 52 | 60 | 58 |

26. Geometry Which can be modeled by light beaming from a lighthouse?
A Point
C Ray
B Plane
D Segment
27. Estimation What is the approximate mean height of the 7 tallest peaks in Texas listed below?
$\left.\begin{array}{|l|l|}\hline \text { Peaks in Texas } & \text { Height in Feet } \\ \hline \text { Guadalupe Peak } & \vdots\end{array}\right) 8,749$

## Median, Mode, and Range

## How can data be described by one number?

Trey listed, in order, the playing times for the best-selling CD of each music type.

How can he describe the data with one number?

## CD Playing Times

Minutes: Music Type

| 59 | Ol | Popular |
| :--- | :--- | :--- |
| 61 | $\vdots$ | Country |
| 63 | $\vdots$ | Blues |
| 63 | $\vdots$ | Soundtrack |
| 64 | $\vdots$ | Gospel |
| 67 | $\vdots$ | Jazz |
| 72 | $\vdots$ | Classical |

## Guided Practice*

## Do you know HOW?

In 1 through 3, identify the median, mode, and range for each set of data.

1. $5,7,5,4,6,3,5$
2. $21,21,23,32,43$
3. $13,14,14,16,17,19$

For an even number of values, the median is the number halfway between the two middle values.

## Do you UNDERSTAND?

4. What operation is used to find the range?
5. In the example at the top, how would the median and mode change if the playing time for the Blues CD changed to 61 minutes?
6. What would the range of playing times be if the 72-minute CD was removed from the list?

## Independent Practice

In 7 through 9, use the table at the right.
7. What are the median, mode, and range for the data?
8. What would happen to the range if the temperature were $82^{\circ}$ F on Monday?
9. If the data for Friday were removed from the table, what would the median, mode, and range be?

| 5-day Weather Forecast |  |  |
| :---: | :---: | :---: |
| Day | $\vdots$ | Temperature |
| Monday | $\vdots$ | $80^{\circ} \mathrm{F}$ |
| Tuesday | $\vdots$ | $80^{\circ} \mathrm{F}$ |
| Wednesday | $\vdots$ | $82^{\circ} \mathrm{F}$ |
| Thursday | $\vdots$ | $84^{\circ} \mathrm{F}$ |
| Friday | $\vdots$ | $78^{\circ} \mathrm{F}$ |

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## Find the median.

List the data from least to greatest.
$59,61,63,63,64,67,72$
Identify the median, or the middle data value in an odd numbered, ordered set of data.

The median of the number of minutes of playing time is 63 .

## Find the mode.

59, 61, 63, 63, 64, 67, 72
Identify the mode, or the data value that occurs most often in the data set.

The mode of the number of minutes of playing time is 63.

## Find the range.

59, 61, 63, 63, 64, 67, 72
Identify the range, or the difference between the greatest and least values.
$72-59=13$
The range of the number of minutes of playing time is 13 .

## Problem Solving

10. Ricardo kept a record of the 7 hottest days of the summer. Use the list below to find the median, mode, and range of the temperatures.
```
98}\mp@subsup{}{}{\circ}\textrm{F}\quad10\mp@subsup{2}{}{\circ}\textrm{F}\quad10\mp@subsup{0}{}{\circ}\textrm{F}\quad99\mp@subsup{}{}{\circ}\textrm{F
103'F 98* F 101'F
```

12. Reasoning For each statistical measure (mean, median, mode, and range) tell whether that number is always, sometimes, or never one of the numbers in the data set.

For 14 through 17, use the table.
14. What was the median number of visitors to the Statue of Liberty from May through September in 2005?
15. What is the range of the data?
16. How many months had over 500,000 visitors?
17. Writing to Explain Why do you suppose there had been many fewer visitors in September, than in July or August?
11. Writing to Explain How can you tell the difference between the net for a triangular prism and the net for a triangular pyramid?
13. Think About the Process One side of a rectangular garden is 13 feet and the other side is 3 feet. Which expression shows how to find the perimeter?
A $(2 \times 13)+(2 \times 3)$
C $2 \times 13 \times 3$
B $13 \times 3$
D $3+13$

| Visitors to the Statue of Liberty |  |  |
| :--- | :---: | :---: |
| 2005 | $\vdots$ | Visitors |
| May | $\vdots$ | 430,235 |
| June | $\vdots$ | 492,078 |
| July | $\vdots$ | 589,166 |
| August | $\vdots$ | 542,292 |
| September | $\vdots$ | 367,441 |

SDAP 3.1, Crade 6 En-m Represent all possible outcomes for compound events in an organized way (e.g, tables, grids, tree diagrams) and express the theoretical probability of each outcome. Also MR 3.3

## Outcomes

## How can tree diagrams help you list possible outcomes?

How many outcomes are possible when spinning a spinner once and then tossing a coin twice? Use a tree diagram to list all possible outcomes. A tree diagram is a diagram used to organize outcomes of an experiment.


## Guided Practice*

## Do you know HOW?

In 1 through 3, list the possible outcomes.

1. Tossing a number cube
2. Spinning a spinner divided into white, blue, black, and purple
3. Tossing an even number on a number cube

## Do you UNDERSTAND?

4. Writing to Explain In the example above, how would your tree diagram change if the spinner had 4 colors? How many possible outcomes would there be?
5. Write a multiplication equation to find the possible outcomes of tossing a number cube and spinning a spinner with 4 different colors.

## Independent Practice

6. Two spinners are spun. Copy and complete the tree diagram to show the possible outcomes.


Spinner 1 Spinner 2 Outcome
7. Josh and Susan are running for class president. Mark, Maria, Lee, and Eva are running for vice-president. How many possible outcomes are there for electing a president and a vice president?


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A tree diagram shows the sample space, which is the set of all possible outcomes.
 You can also find the number of outcomes by multiplying.


## Problem Solving

8. On four tests, Justin scored 90, 85, 90, and 95 . What is the mean score of the four tests?
9. John, Andy, and Miguel run in the first race. Sharon, Marie, and Mona run in the second race. How many different outcomes are possible for the winning pairs?
A 3
B 6
C 9
D 12
10. Writing to Explain How does a tree diagram make it easier to tell which outcome occurs most often?
11. Think About the Process Jennifer's paycheck was $\$ 314.79$. She used $\$ 205.75$ of that money to pay bills. Then she spent $\$ 58$ on groceries. Which expression shows how to find how much money Jennifer has left?

A $\$ 314.79+\$ 205.75-\$ 58$
B \$314.79-\$205.75-\$58
C $\$ 314.79-\$ 205.75+\$ 58$
D $\$ 314.79+\$ 205.75+\$ 58$
9. Steve's dog is 10 lb heavier than Marsha's dog. Together, the dogs weigh 42 lb . How much does each dog weigh?
11. Surfers in California often have a variety of wetsuits and surfboards. If a surfer has 3 wetsuits and 6 surfboards, how many different combinations of a wetsuit and a surfboard could the surfer have?
A 6
B 9
C 18
D 36
13. Lara's ice skating lesson started at 11:15 A.M. and ended at 12:50 P.м. How long did the lesson last?
15. How many different outfits consisting of one pair of jeans, one T-shirt, and one jacket can you make if you have three pairs of jeans, four T-shirts, and two jackets to choose from?
16. Algebra Find the value of $n$, if $n \times 400=28,000$.

SDAP 1.3 Use fractions and percentages to compare data sets of different sizes.

## Writing Probability as a Fraction

What is the probability of an event?
Reuben writes each letter of his name on a separate piece of paper and puts them in a bag. He chooses one piece of paper from the bag without looking.
The probability of an event is a number that describes the chance the event will occur.

## Another Example What is the probability of two events

 happening together?Eva puts the letters of her name into a bag and chooses a letter out of the bag without looking. She puts the letter back into the bag and chooses again without looking. What is the probability that Eva chooses an A both times?

Draw a tree diagram.
$\begin{array}{ll}\text { First } & \begin{array}{l}\text { Second } \\ \text { Letter } \\ \text { Letter }\end{array} \text { Outcome }\end{array}$


Find the probability.
There are a total of 9 possible outcomes when the two letters are chosen and the first is replaced. One of the outcomes is favorable because only one of the outcomes has an A both times.

The probabilty of any event ranges from 0 to 1 .

$P(A, A)=\frac{1}{9}$
The probability that Eva chooses an A both times is $\frac{1}{9}$.

## Gxplain 17

1. Use the tree diagram above to find the probability of choosing the same letter twice. Hint: Look for pairs of outcomes with the same letter.
2. If an event has a probability of $\frac{3}{4}$, is it less likely, more likely, equally likely, impossible, or certain to occur?

Probability of an event $=\frac{\text { number of favorable outcomes }}{\text { total number of possible outcomes }}$
What is the probability that Reuben will choose the letter B?

There is 1 favorable outcome out of 6 possible outcomes, R, E, U, B, E, or N. The outcomes are equally likely (have the same chance of occurring). The probability of choosing the letter B can be written as $P(B)$.
$P(B)=\frac{1}{6}$
The probability that Reuben chooses a B out of the bag is $\frac{1}{6}$.

What is the probability that Reuben will choose the letter $E$ ?

There are 2 favorable outcomes out of 6 possible outcomes (since E appears twice). $P(\mathrm{E})=\frac{2}{6}=\frac{1}{3}$
The probability that Reuben chooses an $E$ out of the bag is $\frac{1}{3}$ or $P(E)=\frac{1}{3}$.

## Guided Practice*

## Do you know HOW?

For 1 through 4, use the spinner shown.

1. Find $P$ (blue).
2. Find $P$ (yellow).
3. Find $P$ (red).

4. Find $P$ (green).

## Do you UNDERSTAND?

5. Writing to Explain In the example above, is it likely, unlikely, impossible, or certain that Reuben draws a B?
6. What is the probability that Reuben will NOT draw an R?

## Independent Practice

7. Write each letter of your first name on a separate small sheet of paper. Put each piece into a box. Do an experiment where you pick one letter and replace it each time. Do this 10 times. Record the number of times you pick each letter, and then write the probability as a fraction.

For 8 through 12, suppose you toss a quarter and a penny.
8. Make a tree diagram to show the possible outcomes.
9. Find $P$ (one head).
10. Find $P$ (two heads).
11. Find $P$ (quarter heads, penny tails)
12. Find $P$ (no heads)


## Independent Practice

13. When a number cube is tossed, there are 6 possible outcomes
$(1,2,3,4,5$, or 6$)$. If the cube is tossed twice and the outcomes are added, the possible sums are from 2 through 12.

Copy the table and give the probability of each sum.

| Sum | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> Occurrences | 1 | 2 | 3 | 4 | 5 | 6 | 5 | 4 | 3 | 2 | 1 |
| Probability | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  |  |  | $\square$ |

14. Which sum (or sums) has the greatest probability of occurring?
15. Which sum (or sums) has the least probability of occurring?

## Problem Solving

16. Geometry Kendra tosses a colored cube. Half of the sides of the cube are red, $\frac{1}{3}$ of the sides are blue, and one side is green. What is the probability that the cube will land on a color other than red when tossed?
17. Think About the Process Jorgé put colored cards into a bag. Two of the cards were green, three were red, one was orange, two were blue, and two were purple. Jorgé wants to find the probability that he will pull an orange card from the bag. What step does Jorgé take to determine the number of possible outcomes?
A Count the number of orange cards.
B Count the number of different colored cards.

C Count the total number of cards in the bag.

D Count the number of cards that are not orange.
17. Mrs. Pierre bought 150 pencils to give to her students. She has three classes with 27,25 , and 23 students each. She wants every student to get the same number of pencils. How many pencils should she give to each student?
19. Carlita buys 3 beanbag throws for \$1.00. What is the probability she will toss one beanbag through a hole in the top row of this game? Assume Carlita always throws a beanbag into a hole.

20. How many parts of each color should there be to make sure that it is equally likely this spinner will land on each of 3 different colors?


## Algebra Connections

## Which Equation is True?

Remember that an equation is a number sentence that uses an equal sign to show that two expressions are equal. Both of the following are equations.
$8+2=10$
$x+25=100$
The first equation is true. You don't know if an algebraic equation is true or false until you replace the variable with a number.

Example: If $m=25$, which equation is true?
$2+m=30 \quad m-20=5$
In the first equation, if you replace $m$ with 25 , the result is $2+25=30$. This equation is false.

In the second equation if you replace $m$ with 25 , the result is $25-20=5$. This equation is true.

If $x=75$, then $x+25=100$ is a true equation.

Decide which equation is true for each replacement of the variable.

1. If $x=20$, which equation is true?

$$
3+x=23 \text { or } \frac{x}{5}=2
$$

2. If $y=100$, which equation is true?

$$
y-80=180 \text { or } y \div 2=50
$$

3. If $x=50$, which equation is true?

$$
x+50=75 \text { or } \frac{x}{10}=5
$$

4. If $x=80$, which equation is true?

$$
40 x=3,200 \text { or } \frac{x}{4}=2
$$

5. If $z=200$, which equation is true?

$$
200-x=200 \text { or } x+100=300
$$

6. If $x=70$, which equation is true?

$$
6 x=420 \text { or } x+7=63
$$

7. If $x=0$, which equation is true?

$$
50 x=50 \quad x+7=7
$$

8. If $x=1$, which equation is true?

$$
40 x=40 \text { or } 40+x=40
$$

9. If a teacher has 32 students and divides them into teams of 4, which equation could be used to find how many students will be on each team? Let $x$ represent the number of students on each team.

$$
\frac{32}{4}=x \quad 4+x=32 \quad \frac{32}{x}=4 \quad 4 \times 32=x
$$

MR 1.2 Determine when and how to break a problem into simpler parts.
Also NS 1.0, MR 2.2.

## Solve a Simpler Problem <br> books from reading list

Owen needs to read 2 books from a list of 6 books. How many different combinations of books are possible?

Follow these steps to solve a simpler problem.

1. Break apart or change the problem into one that is simpler.
2. Solve the simpler problem.
3. Use the answers to the simpler problem to solve the original problem.

## Guided Practice*

## Do you know HOW?

1. Draw a picture to show the number of combinations of pairs for 5 books.
2. Think of extending the table at the top to find the number of pairs with 7 books. What number would you add to 15 to find the number of pairs? How many pairs would there be for 7 books?

## Do you UNDERSTAND?

3. Writing to Explain Is it easier to use the table or to draw a picture as the number of books increases?
4. Write a Problem Write a real-world problem that can be solved by solving a simpler problem.

## Independent Practice

Solve each problem.
5. Continue the pattern in the book problem above. How many pairs of books would there be for 8 books? 9 books? 10 books?
6. Find the number of degrees in a hexagon. HINT: Divide the hexagon into triangles.
a How many triangles are formed?

b How many degrees are in each triangle?
c What is the total number of degrees in the hexagon?
7. Using the same strategy in Problem 6, what is the total number of degrees in a pentagon? An octagon?

## Use letters to represent the books.


2 books: 1 pair

3 books: 3 pairs


4 books: 6 pairs

## Look for a pattern.

Find the pattern. Continue the pattern to 6 books.

| Number of <br> Books | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of <br> Pairs | 1 | 3 | 6 | 10 | 15 |
| +2 |  |  |  |  |  |$\underbrace{}_{+\mathbf{+ 4}_{+5}}$

There are 15 different possible pairs.
8. Juanita tossed 3 number cubes, and these digits came up.


If each of the numbers 2,1 and 4 is used only once to form a 3-digit number, which shows all the possible 3-digit numbers?

A $214,421,142$
B 214, 124, 412, 421
C $214,241,142,412,124$
D 214, 241, 142, 124, 412, 421
10. George was choosing his clothes for the next day. In his closet he had 2 ties, 5 shirts, 3 trousers, and 2 belts. How many different ways could George choose a tie, shirt, trousers, and belt?
12. After seeing a movie, 2 friends stopped for frozen yogurt. Three flavors were available in small, medium, and large sizes. How many different combinations of flavors and sizes are possible?
9. Jill has 3 colored vases to arrange on a shelf.


Let B stand for blue, O for orange, and $G$ for green. Which list shows all the possible arrangements of the vases?
A BOG, OBG, GOB
B BOG, OBG, GOB, GBO
C BOG, OBG, GOB, GBO, OGB
D BOG, OBG, GOB, GBO, OGB, BGO
11. The McMillan family wanted to buy a new vehicle. They could choose a van or a car; a black, silver, or white exterior; and a tan or black interior. How many different vehicles can they buy?
13. Algebra Draw a picture and write an equation to solve.
Niko had $17 \frac{1}{3} \mathrm{ft}$ of fencing. He uses $5 \frac{2}{3} \mathrm{ft}$ to finish a job. How many feet of fencing does Niko have now? Let $f=$ feet of fencing left.

1. The histogram shows the results from a survey asking people how many plays they have seen in the last year. How many people have seen more than 3 plays? (19-2)

Going to the Theater


A 26
B 15
C 11
D 8
2. The music teacher is choosing two students out of 10 to sing a duet in the musical. How many different pairs of students must try out if she wants to try every possible pair? (19-9)

A 50
B 45
C 20
D 12
3. James is drawing the picture graph below to show the number of license plates from each state he saw on a recent trip. He saw 15 Arizona license plates. Which picture should he draw for Arizona? (19-1)

## License Plates James Saw

State Number of License Plates
Nevada 500 WM
Oklahoma FOOCNM FOOMW
New Mexico F 5000 Wm
Arizona
Arkansas
XiOO-MN

Other States 5

$$
\text { Fomow }=5 \text { license plates }
$$

A 500 WN W00

C F 500 WW W

4. The shoe sizes of the starting players on the girls' basketball team are listed below. What is the median of these numbers? (19-6)

7, 6, 5, 6, 8
A 5
B 6
C 7
D 8
5. According to the circle graph below, about what fraction of the $t$-shirts sold were $x$-large? (19-3)


A $\frac{1}{3}$
B $\frac{1}{4}$
C $\frac{1}{5}$
D $\frac{1}{6}$
6. The table shows the heights of a group of friends. What is the mean height of the group in inches? (19-5)

| Name | $\vdots$ | Height (in inches) |
| :--- | :---: | :---: |
| Amy | $\vdots$ | 58 |
| Rudy | $\vdots$ | 62 |
| Jessie | $\vdots$ | 55 |
| Pablo | $\vdots$ | 61 |
| Jared | $\vdots$ | 64 |

A 61
B 60
C 59
D 58
7. A college student plans to take biology, algebra, and literature in the morning. Each letter represents the first letter of each class. Which list below shows all the different orders that these three classes can be taken? (19-7)

A BAL, BAB, LAB
B BAL, LAB, ABL, LBA
C BAL, BLA, ALB, LAB, ABL
D BAL, BLA, ALB, ABL, LAB, LBA
8. The cards shown are placed in a bag and one is drawn without looking. What is the probability Bobby's name is drawn? (19-8)


A $\frac{1}{8}$
B $\frac{2}{8}$
C $\frac{3}{8}$
D $\frac{3}{5}$
9. An airline company is making a graph to display the number of each type of ticket they have available on a flight including first class, business class and economy. Which type of graph would be best to display the data? (19-4)

A Bar or picture graph
B Line plot
C Circle graph
D Line graph

Set A, pages 420-422

Students were asked to name their favorite seasons. Make a bar graph to show the results.

| Favorite Season |  |  |
| :--- | :--- | :--- |
| Summer | $\vdots$ | 8 |
| Autumn | $\vdots$ | 6 |
| Winter | $\vdots$ | 4 |
| Spring | $\vdots$ | 6 |

Favorite Season


Remember that a bar graph or a picture graph can be used to compare data that shows how many or how much. In a picture graph, each symbol represents a certain amount in the data.

1. Students were asked to name their favorite flower. Make a bar graph to show the results.

| Favorite Flower |  |  |
| :--- | :---: | :---: |
| Rose | 10 |  |
| Lily | $\vdots$ | 3 |
| Carnation | $\vdots$ | 3 |
| Tulip | $\vdots$ | 6 |
| Daffodil | $\vdots$ |  |

Remember that histograms are a type of bar graph that has no space between the bars and the bars show equal intervals.

Use the histogram at the left to answer the following questions.

1. How many students were surveyed?
2. How many students had bought more than 5 books?
3. What fraction of the students bought 9,10 , or 11 books?

Using the histogram, find the number of students who purchased 3-5 books. The bar with the 3-5 interval has a height of 8 . The number of students is 8 .

Set C, pages 426-428

A group of 100 students were asked to name their favorite type of television program. Make a circle graph to show the results.

| Favorite Type of Television Program |  |  |
| :--- | :--- | :--- |
| Comedy | $\vdots$ | 25 |
| Sports | $\vdots$ | 50 |
| Drama | $\vdots$ | 10 |
| Other | $\vdots$ | 15 |

Comedy $=25$ out of $100=\frac{1}{4}$.
Sports $=50$ out of $100=\frac{1}{2}$,
Drama $=10$ out of $100=\frac{1}{10}$.
Other $=\frac{15}{100}=\frac{3}{20}$.


Draw a circle with sectors for
$\frac{1}{2}, \frac{1}{4}$, and $\frac{1}{10}$ and label those sectors
with the program type. The remaining sector shows $\frac{3}{20}$ or Other.

Remember a circle graph shows the whole amount (100\%) and each sector represents a part of the whole amount.

Jill spent a total of 30 hours exercising last month: Jogging: 15 hours; Cycling: 10 hours; Swimming: 5 hours.


1. Copy the circle graph and label each sector with the activity and fraction.
2. What fraction represents the part of the day Jill spent cycling?

Set D, pages 430-431

Students were asked to name their favorite animal. Make a bar graph to show the results.


Favorite Animal


Remember that making a graph makes it easy to visualize data and answer questions about the data.

| Monthly Snowfall |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Month | Dec. | Jan. | Feb. | Mar. |
| Calories | 6 | 12 | 18 | 4 |

1. Make a line graph of the data.
2. Which month has a snowfall 3 times as great as December?

Set E, pages 432-433

Find the mean of this data set:
$2,3,8,3,5,6,1$
The mean is an average. To find the mean, add the data and divide by the number of data.
$(2+3+8+3+5+6+1) \div 7=28 \div 7=4$
The mean is 4 .

Remember that the mean is an average. You must add the data and divide by the number of addends.

1. $5,4,1,3,1,10$
2. $6,9,5,2,8$
3. $4,2,2,5,3,3,2$

Set $F$, pages 434-435

Find the median, mode, and range for this set of data: 10, 15, 20, 12, 10

To find the median, list the data in order from least to greatest and find the middle value.
$10,10,12,15,20 \quad$ The median is 12.
To find the mode, list the data in order from least to greatest and find the value that occurs most often.
$10,10,12,15,20 \quad$ The mode is 10.
To find the range, subtract the least value from the greatest value.
$20-10=10 \quad$ The range is 10.

Remember that if there is an even number of data, you must add the two middle numbers when the data are ordered from least to greatest and divide by 2 to find the median.

1. Find the median of this data set:

$$
\begin{array}{llll}
27.5 & 27 & 30 & 29.5
\end{array}
$$

2. Find the mode of this data set: $12,6,9,5,8,12,8,1,4,12,6$
3. Find the range of this data set: $87,84,90,75,100,88$

Set G, pages 436-437

Judy can pick either a hamburger or a chicken sandwich and then either a coleslaw, potato salad, or a fruit salad. How many different lunch combinations can she have?

Draw a tree diagram.


There are 6 possible combinations, so Judy has a choice of 6 different lunches.

Remember that when you make a tree diagram, start with 1 item and list all the possible combinations with it. Continue with all the items.

1. Make a tree diagram to show the sample space for tossing a number cube and then tossing a quarter.
2. Make a tree diagram to show the results of choosing among tuna, cheese, and turkey sandwiches, and milk and juice drinks.

You spin the spinner once. Find $P$ (landing in a green section).


Probability $=\frac{\text { number of favorable outcomes }}{\text { number of possible outcomes }}$
So, $P($ green $)=\frac{3}{6}=\frac{1}{2}$

Remember that when you express a probability as a fraction, always write the fraction in simplest form.

Use the letters in MATHEMATICS. Each letter is written on a separate piece of paper and the pieces are put in a bag.

Find the probability of choosing each letter or letters out of the bag.

1. $P$ (a consonant)
2. $P$ (the letter $A$ )
3. $P$ (not the letter $A$ )

Set I, pages 442-443

To solve a simpler problem, follow these steps:

## Step 1

Break apart or change the problem into one that is simpler to solve.

## Step 2

Draw a picture or use objects to look for a pattern to solve the simpler problem.

## Step 3

Use the answers to the simpler problem to solve the original problem.

Four people shake hands with each other once. How many handshakes are there in all?

Use A, B, C, D to represent the people.
AB, AC, AD
$B C, B D, C D$
There are 6 handshakes.

Remember that you can draw a picture or use objects to look for a pattern in finding the relationship between the simpler problem and the original problem.

1. How many different teams of 2 people can be chosen from 4 people?
2. How many different teams of 3 people can be chosen from 5 people?
3. What is the sum of the angles of an octagon?
