4

The Leaning Tower of Pisa is located in Pisa, Italy, and is one of the most famous buildings in the world. How can you construct an angle that is congruent to the angle that the tower leans? You will find out in Lesson 20-1.

Review What You Know!

Vocabulary

Choose the best term from the box.

- line
- line segment
- perpendicular
- ray

1. A ? extends in two directions infinitely.

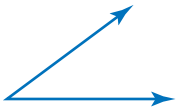
2. A ? has one endpoint and extends in one direction, while a ? has two endpoints.

3. ? lines intersect at one point and create four right angles.

Identifying Angles

Classify each angle as acute, obtuse, straight or right.

4.



5.

6.

7.

Patterns

Find the next two numbers in each pattern.

8. 1.5, 1.7, 1.9, 2.1, 2.3, ▢, ▢

9. 26, 21, 16, 11, ▢, ▢

Shapes

Writing to Explain Write an answer for each question.

10. Can a triangle have two obtuse angles?

11. Is a square always a rectangle?

Lesson
20-1

MG 2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).

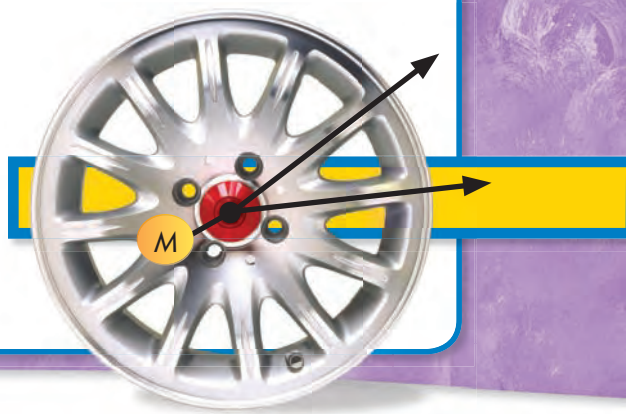
Constructing Angles

Hands-On GeoTool

How can you construct an angle congruent to another angle?

A geometric construction is the drawing of a figure using only a compass and a straightedge. A ruler and a protractor are not used in making a construction.

How would you construct $\angle B$ congruent to $\angle M$?

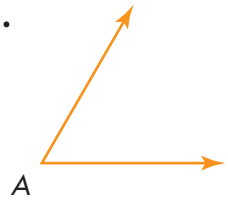


Guided Practice*

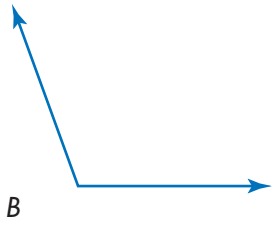
Do you know HOW?

In **1** and **2**, trace each angle on a sheet of paper. Then construct an angle congruent to each given angle.

1.



2.



Do you UNDERSTAND?

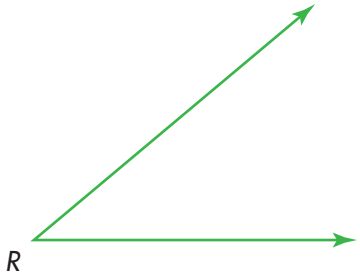
3. **Writing to Explain** Why is it important to keep the same compass setting in Step 1?

4. How can you tell if a constructed angle is congruent to the original angle?

Independent Practice

In **5** through **7**, trace each angle on a sheet of paper. Then construct an angle congruent to each given angle.

5.



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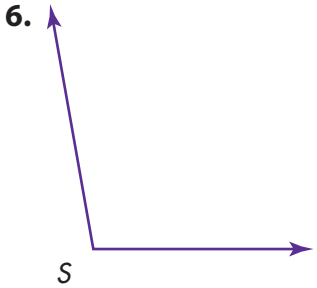

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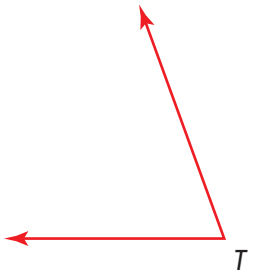

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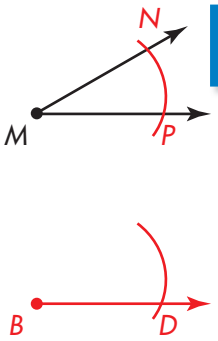
*For another example, see Set A on page 466.

Step 1

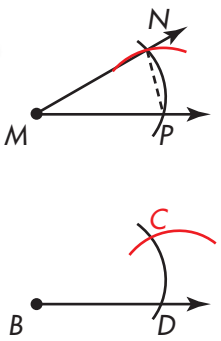
Draw a ray with endpoint B. With point M as center, use your compass to draw an arc that intersects both sides of $\angle M$. Label the points of intersection N and P.

With the same compass setting and endpoint B as center, draw an arc that intersects the ray at point D.



Step 2

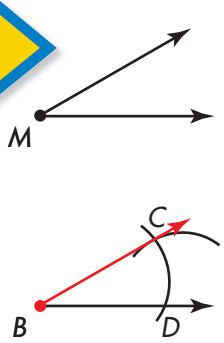
Set the compass to the length $\overline{NP}$. Then, with point D as center, draw an intersecting arc. Label the point C.



Step 3

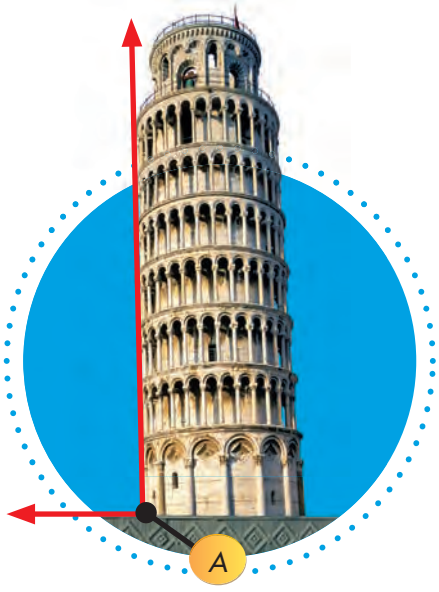
Using a straightedge, draw ray BC.

$\angle B \cong \angle M$



Problem Solving

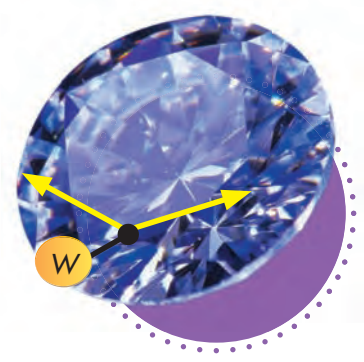
8. The Leaning Tower of Pisa is located in Italy. It leans to the south at the angle shown in the picture below. Construct an angle congruent to $\angle A$.



9. Mario has saved a number of quarters, nickels, dimes, and pennies. He will use \$3.93 to buy a new toy for his dog. Using the fewest coins, what combination of coins will equal \$3.93?

10. Justin climbed 15 steps to the second floor of a building. Each step is 8 inches high. How many feet higher is the second floor than the first?

11. Diamond cutters must cut angles precisely so that the greatest amount of light is reflected from the facets. Refer to the photo of the diamond below. Construct an angle congruent to $\angle W$.



12. The high temperatures in a city for 7 days were 87°, 87°, 90°, 90°, 90°, 92°, and 87°. What was the mean temperature for the 7 days?

A 87° **C** 90°

B 89° **D** 91°

Lesson
20-2

MG 2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).

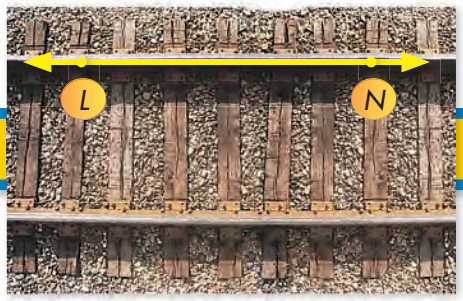
Constructing Lines

Hands-On GeoTool

How can you construct perpendicular and parallel lines?

The rails of the track are parallel. The ties are perpendicular to the rails.

Construct a line perpendicular to $\overleftrightarrow{LN}$ and a line parallel to $\overleftrightarrow{LN}$.



Another Example

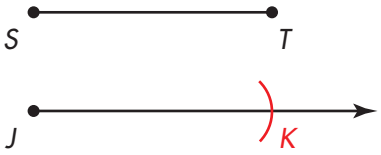
How do you construct a line segment congruent to a given line segment?

Without measuring with a ruler, draw $\overline{JK}$ congruent to $\overline{ST}$.

Step 1 Draw a ray. Label the endpoint *J*.

Step 2 On $\overline{ST}$, with point *S* as the center, open the compass so that it lines up with point *T*.

Then place the compass on the ray with point *J* as the center. Without changing the compass setting, draw an arc that intersects the ray. Label the point of intersection *K*. $\overline{JK}$ is congruent to $\overline{ST}$.

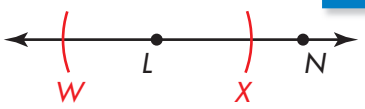


Explain It

1. How is constructing a figure different from drawing a figure?
2. In Step 2, could the ray be any length?
3. Does a line segment need to be horizontal in order to construct another segment congruent to it?

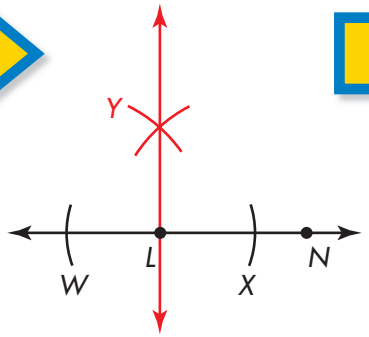
Step 1

Draw a line with points L and N. With L as center, draw two arcs that intersect $\overleftrightarrow{LN}$. Label the points W and X.



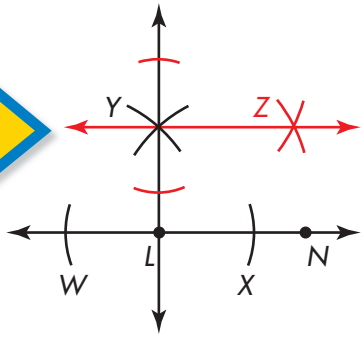
Step 2

Set the compass wider. Using W and X as centers, draw arcs that intersect. Label the point Y. Draw $\overleftrightarrow{LY}$.



Step 3

Repeat Steps 1 and 2 at Y to find point Z.



$\overleftrightarrow{LY} \perp \overleftrightarrow{LN}$ and $\overleftrightarrow{LN} \parallel \overleftrightarrow{YZ}$

Guided Practice*

Do you know HOW?

1. Draw a line perpendicular to line CD. Copy $\overleftrightarrow{CD}$ on a separate sheet of paper and construct $\overleftrightarrow{TC}$ so that it is perpendicular to $\overleftrightarrow{CD}$.

2. Draw a segment congruent to $\overline{EF}$. Copy $\overline{EF}$ on a separate sheet of paper. Then draw a ray labeled $\overrightarrow{MN}$. On that ray, construct $\overline{MP}$ so that $\overline{MP}$ is congruent to $\overline{EF}$.

Do you UNDERSTAND?

3. **Writing to Explain** In Step 2 of the example above, why is it necessary to set the compass wider than the length of segment WL?

4. Look at the line TC you constructed in Problem 1 that is perpendicular to line CD. Use point T and construct a line perpendicular to line TC. How is that line related to line CD?

Independent Practice

In **5** through **7**, copy the figures on a separate sheet of paper and follow the directions.

5. Construct a line perpendicular to line XY.

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*For another example, see Set B on page 466.

Independent Practice

6. Construct a line perpendicular to line *MN*. Then construct a line through it that is parallel to line *MN*.

7. Draw a line segment that is congruent to $\overline{CD}$.

Problem Solving

Engineers are making plans to lay new railroad tracks between cities. In **8** and **9**, use the information in the table.

Data

	Los Angeles	San Diego	Sacramento
San Francisco	384 mi	502 mi	87 mi
Eureka	647 mi	766 mi	309 mi

8. If it costs $155 per mile to construct railroad tracks, how much would it cost to build tracks from San Francisco to Los Angeles?

9. **Estimation** Railroad ties are set about 2 feet apart. About how many railroad ties are there between Eureka and Sacramento?

Tip *1 mile = 5,280 feet*

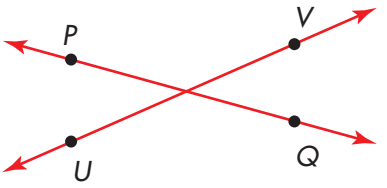
10. If you are comparing two negative integers on a number line, how can you tell which one is greater?

11. Four friends played golf. Their scores were $^{+}3$, $^{-}1$, $^{+}4$, and $^{-}4$ in relation to par. The least score wins. Arrange the scores from best to worst.

12. **Algebra** Ted has 15 trophies. This is 5 times as many as Harold has. How many trophies does Harold have? Write and solve an equation to answer the question.

13. **Writing to Explain** Explain how perpendicular lines are similar to intersecting lines.

14. Use the figure to tell whether the statements are true or false.



a $\overline{PQ}$ is parallel to $\overline{UV}$

b $\overline{PQ}$ intersects $\overline{UV}$

c $\overline{PQ}$ is perpendicular to $\overline{UV}$

15. **Estimation** The gas tank in Shondra's car can hold 18 gallons. Her car gets about 22 miles per gallon of gas. On a recent trip, Shondra used about 12 gallons of gas. Which is the best estimate of the distance Shondra drove?

A 120 miles

B 200 miles

C 400 miles

D 1,200 miles

Find each difference. Simplify if possible.

1. $5\frac{5}{8} - 3$

2. $2\frac{2}{3} - 1\frac{5}{6}$

3. $3\frac{1}{4} - \frac{1}{8}$

4. $7 - 3\frac{5}{12}$

5. $6\frac{1}{2} - 2\frac{1}{2}$

6. $5\frac{1}{3} - 3\frac{5}{12}$

7. $8\frac{1}{6} - 2\frac{1}{4}$

8. $4\frac{4}{5} - 1\frac{3}{15}$

Find each product. Estimate to check if the answer is reasonable.

9. 36×36

10. 379×39

11. 405×19

12. 564×50

13. $2{,}705 \times 30$

14. $9{,}191 \times 19$

15. 787×211

16. 904×508

17. 759×196

18. 999×333

Error Search Find each answer that is not correct. Write it correctly and explain the error.

19. $10{,}000 - 5{,}831 = 5{,}169$

20. $12.5 \times 0.75 = 93.75$

21. $14{,}976 + 13{,}867 = 28{,}743$

22. $9\overline{)9.27} = 1.03$

23. $36\overline{)3{,}637} = 101$

Number Sense

Estimating and Reasoning Write whether each statement is true or false. Explain your reasoning.

24. The quotient of $1{,}546 \div 5$ is less than 300.

25. The product of 9.32 and 4.7 is less than 36.

26. The difference of 6,631 and 3,021 is greater than 2,000 and less than 4,000.

27. The sum of $43.04 + 21.56$ is 0.04 more than 64.56.

28. When $x = {}^{-}9$, the expression $x + {}^{+}5$ equals ${}^{-}14$.

29. The expression $(4\frac{5}{6} + 3\frac{1}{2} \times \frac{3}{4}) \times 0$ equals 0.

Lesson
20-3

MG 2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).

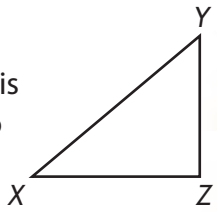
Constructing Shapes

Hands-On GeoTool

How can you construct congruent triangles?

Triangles are rigid figures which makes them useful in constructing buildings.

Construct a triangle that is congruent to triangle *XYZ*.

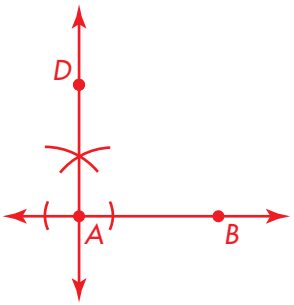


Another Example How can you construct a rectangle?

A rectangle has four right angles and opposite sides that are parallel and congruent. Construct a rectangle *ABCD*.

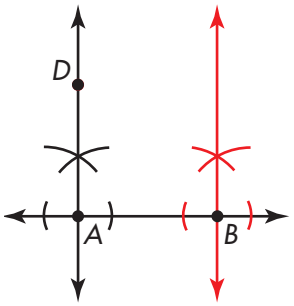
Step 1

Draw a line *AB*. Construct a line perpendicular to line *AB* at point *A*. Choose a point *D* on the line.



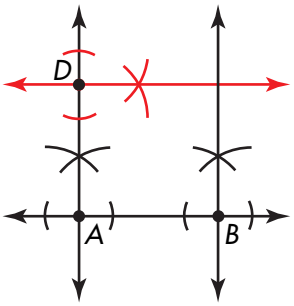
Step 2

Construct a line perpendicular to line *AB* at point *B*.



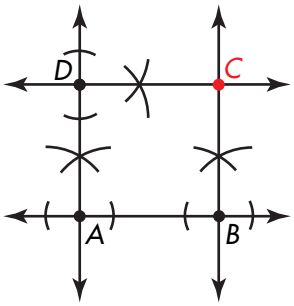
Step 3

Construct a line perpendicular to line *AD* at point *D*.



Step 4

The point where the perpendicular lines you constructed in Steps 2 and 3 intersect is point *C*.

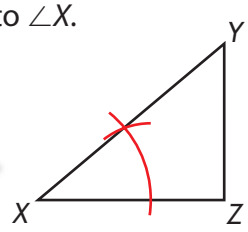


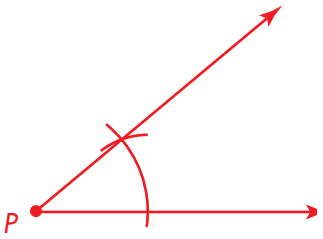
Explain It

1. What are the the four right angles in *ABCD*?
2. Name the two pairs of parallel and congruent sides in *ABCD*.

Step 1

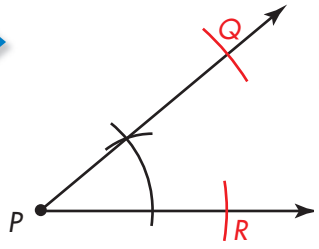
Construct $\angle P$ congruent to $\angle X$.





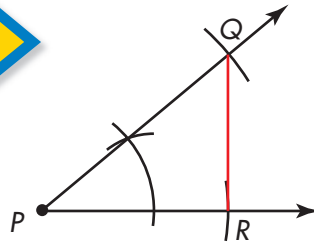
Step 2

On one side of $\angle P$, construct $\overline{PQ} \cong \overline{XY}$.
On the other side of $\angle P$, construct $\overline{PR} \cong \overline{XZ}$



Step 3

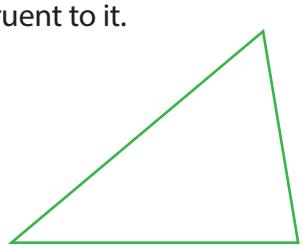
Draw segment QR.
$\triangle PQR \cong \triangle XYZ$



Guided Practice*

Do you know HOW?

1. Copy the following triangle on another sheet of paper. Then construct a triangle congruent to it.



2. Construct a rectangle using a compass and straightedge.

Do you UNDERSTAND?

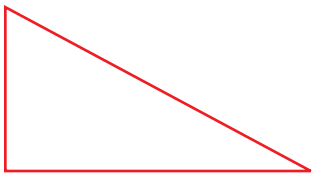
3. When you construct a rectangle, what is the measure of each angle?

4. How can you be sure that a triangle you construct is congruent to the original triangle?

Independent Practice

Copy each triangle on another sheet of paper. Then construct a triangle congruent to it.

5.



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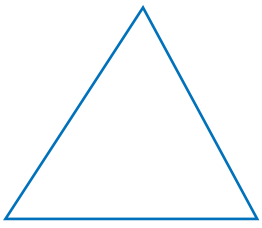

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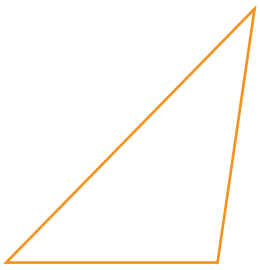

8. Copy the following line on another sheet of paper. Use it to construct a rectangle.

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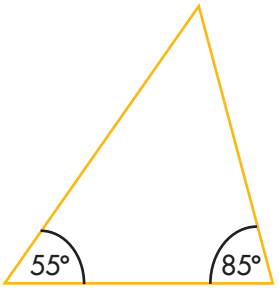
**For another example, see Set C on page 467.*

Independent Practice

9. Ellen studied math for $1\frac{1}{2}$ hours, science for $\frac{3}{4}$ hour, and history for $\frac{3}{4}$ hour. How many hours did she study in all?

10. Alan bought 2 packages of tennis balls for $7.98 each. How much change will Alan get from $20?

11. **Geometry** What is the measure of the third angle in the triangle?



12. **Algebra** Yasmin is 6 inches taller than Burt. If b represents Burt's height, which expression represents Yasmin's height?

A $b + 6$

B $b \times 6$

C $b - 6$

D $b \div 6$

13. **Reasoning** Terry is arranging furniture and wants to center her 6.5-foot table against a wall that measures 15 feet. How far will the table be from each end of the wall? You can draw a picture to help you.

14. Squares for a quilt are being cut from a piece of material that is 15 inches wide and 20 inches long. The squares are 4 inches on each side. How many whole squares can be cut from the material? Draw a picture to help you.

15. Mr. Smith is redecorating his dining room. Tell if he needs to find the perimeter or area for each the following.

a the amount of wallpaper border to go around the top of the room

b the amount of carpeting needed to cover the floor

16. **Writing to Explain** How can you construct a triangle that is congruent to triangle *EFG* formed by the roofline of the house?

17. Let x represent the number of miles Gina ran. Paul ran 3 more than $\frac{1}{2}$ as many miles as Gina. Which expression represents the distance Paul ran?

A $\frac{1}{2}(x + 3)$

B $\frac{1}{2}x + 3$

C $3x + \frac{1}{2}$

D $2(x + 3)$

Mixed Problem Solving

Math and Science

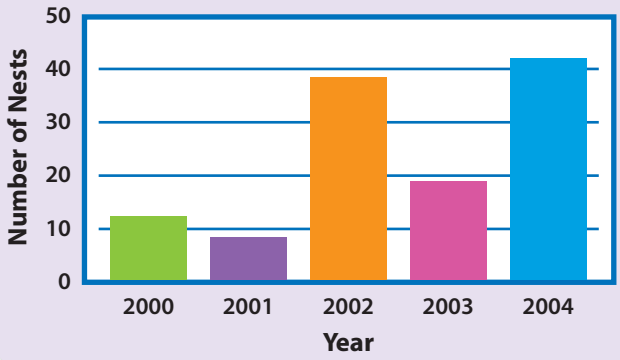
1. How many Kemp's Ridley sea turtle nests were found in 2002?

2. How many more Kemp's Ridley nests were found in 2004 than in 2001?

3. Is there a trend in the data for Kemp's Ridley sea turtle nests? Explain.

4. Between which two years did the number of Kemp's Ridley nests increase the most?

Kemp's Ridley Nests on Padre Island



5. On a picture graph that shows the number of participants in rubber duck derbies, one picture equals one thousand participants. How many participants are represented by 3.5 pictures?

6. **Strategy Focus** Solve using the strategy Use Reasoning.

   Greg, Rick, and Tom like either math, science, or art best. Tom dislikes art. Rick is not the student who likes art or math best. Which student likes each subject best?

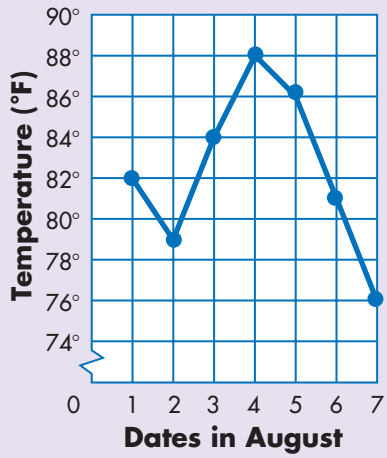
7. What was the high temperature on August 4?

8. On which date was the high temperature the lowest?

9. What was the difference between the greatest and least high temperatures for the week?

10. A scientist is studying the types and number of plants in a small area. What kind of graph should the scientist use to present his data?

High Temperatures in Chicago



Lesson
20-4

MR 2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. Also **MR 2.6, MG 2.0**

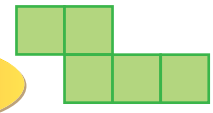
Problem Solving

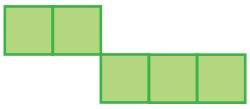
Use Objects

Hands-On
square tiles

A pentomino is an arrangement of 5 identical squares in a plane. The squares must be attached to one another edge to edge.

This is a pentomino.





This is not a pentomino.

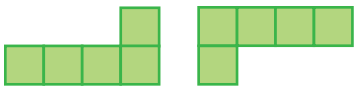
Using 5 identical square tiles, how can you build 3 more pentominoes that have 3 squares in a row?

Guided Practice*

Do you know HOW?

1. Is the following a pentomino? Explain.

2. Are these two pentominoes the same or different? Explain.



Do you UNDERSTAND?

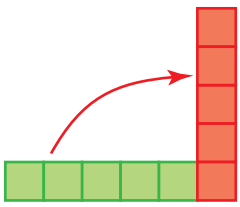
3. In the example above, how many more pentominoes can you find with 3 in a row?

4. **Write a Problem** Write a real-world problem that can be solved by using objects.

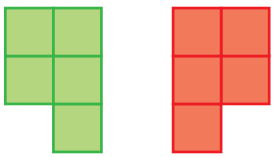
Independent Practice

In **5** and **6**, tell whether the pentominoes in each pair are related by a reflection or a rotation.

5.



6.



In **7** and **8**, use objects to help you solve the problem.

7. How many pentominoes can you build with 5 in a row?

8. How many pentominoes can you build with 4 in a row?

Stuck? Try this....

- What do I know?
- What am I asked to find?
- What diagram can I use to help understand the problem?
- Can I use addition, subtraction, multiplication, or division?
- Is all of my work correct?
- Did I answer the right question?
- Is my answer reasonable?

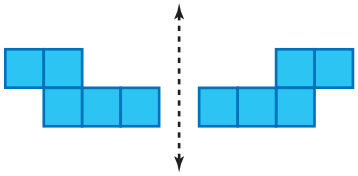
*For another example, Set D on page 467.

Read and Understand

What am I asked to find?

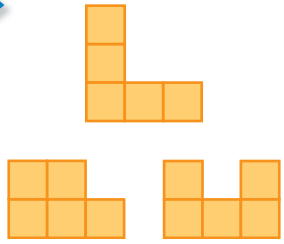
Three more unique pentominoes that have 3 squares in a row

Two pentominoes are the same if they can be matched together by rotating or reflecting.



Plan and Solve

I can use objects to build 3 more pentominoes with 3 in a row.



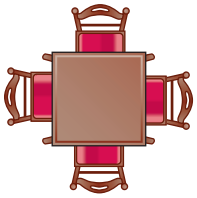
Here are 3 possible solutions.

Look Back and Check

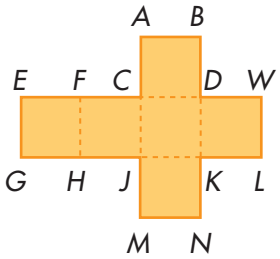
- All are attached edge to edge.
- All can be rotated and reflected. None are repeated.

The 3 pentominoes I've made are all unique.

9. Suppose each square in a pentomino is a table that seats one person on a side. Find a table arrangement (a pentomino) that can seat 12 people.



10. The figure below can be folded to form a box. After it is folded, which face will be parallel to Face *ABDC*?



11. Maureen is buying a game that is priced at $15. She has a coupon worth $2.50 off the regular price. After Maureen gives the clerk a $20 bill, how much change does she receive?

12. James and Kurt were paid $176 for landscaping a yard. James worked 9 hours and Kurt worked 13 hours. How much is Kurt's fair share of the earnings? James's share?

13. Use objects to build pentominoes with 2 squares in a row. How many of these kinds of pentominoes can be built?

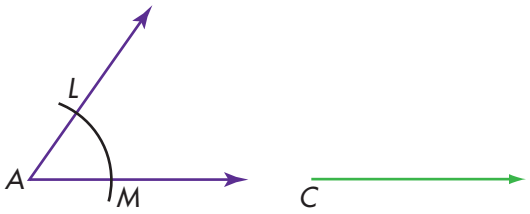
14. Make an organized list. How many different combinations of coins can make $0.42 if one of the coins is a quarter? One possible combination: 1 quarter, 17 pennies.

15. At the concert, Mischa, Jordan, and Elijah are sitting together in a row. Make a list of the possible orders the three could be sitting.

16. **Estimation** A great white shark can weigh 4,400 lbs. A dolphin can weigh 440 lbs. About how many times as heavy is the shark as the dolphin?

DIGITAL Animated Glossary, eTools
www.pearsonsuccessnet.com

1. Teresa is constructing $\angle C$ congruent to $\angle A$ for her drafting class. What is the next step that Teresa should take to complete the construction? (20-1)



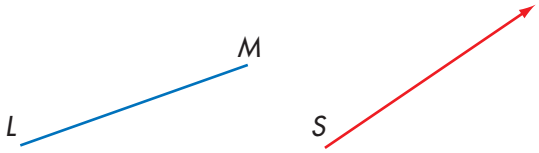
A Measure the opening of arc *LM* with a compass.

B Measure the opening of arc *LM* with a protractor.

C Draw an arc with center at point *C* using the same compass setting used to draw arc *LM*.

D Draw another arc that intersects arc *LM*.

2. Mrs. Bradley is an architect who needs to construct $\overline{ST}$ congruent to $\overline{LM}$. She has drawn a ray with endpoint *S* as shown. What is the next step she should take? (20-2)



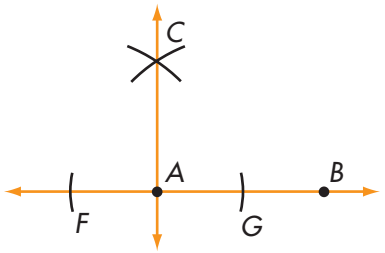
A Draw an arc above $\overline{LM}$.

B Draw an arc above and below $\overline{LM}$.

C Draw an arc on $\overline{LM}$.

D With point *L* as the center, open the compass so it lines up with point *M*.

3. What is the next step in constructing a line parallel to $\overleftrightarrow{AB}$? (20-2)



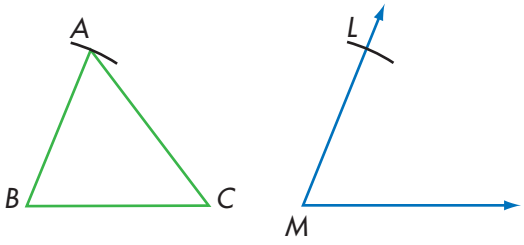
A Draw segment *BC*.

B Construct a line perpendicular to line *AC*.

C Construct a line segment congruent to line segment *AB*.

D Draw a line through point *G*.

4. Angelo is constructing *LMN* so that it is congruent to *ABC*. He has constructed $\angle M$ congruent to $\angle B$ and $\overline{ML}$ congruent to segment $\overline{BA}$.
What is the next step? (20-3)



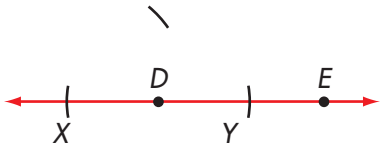
A Measure the length of $\overline{BA}$ with his compass.

B Measure the length of $\overline{BA}$ with his protractor.

C Construct $\overline{MN}$ congruent to $\overline{BC}$.

D Connect point *L* with $\overrightarrow{MN}$.

5. What is missing from the construction of a line perpendicular to $\overleftrightarrow{DE}$? (20-2)



A Arc above $\overleftrightarrow{DE}$ with center at point *E*.

B Arc above $\overleftrightarrow{DE}$ with center at point *D*.

C Arc above $\overleftrightarrow{DE}$ with center at point *Y*.

D A line parallel to $\overleftrightarrow{DE}$.

6. The first step in constructing a rectangle *ABCD* is to draw line *AB*. Which of the following choices could be the next step? (20-3)

A Construct two lines that are parallel.

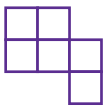
B Construct a line segment that is congruent to segment *AB*.

C Construct two angles that are congruent.

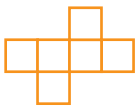
D Construct a line perpendicular to $\overleftrightarrow{AB}$ at point *A*.

7. Suppose each of the 6 squares in a hexamino represents a table which can seat one person on a side. Which arrangement of 6 tables can seat exactly 12 people? (20-4)

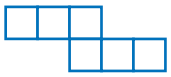
A



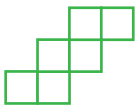
B



C



D



8. What two tools are used to construct geometric figures? (20-1)

A ruler and straightedge

B straightedge and compass

C ruler and protractor

D compass and protractor

9. Aidan is a graphic designer and needs to construct $\angle S$ congruent to $\angle T$ for one of his projects. What is the first step in the construction? (20-1)

A Draw a ray with endpoint *S*.

B Draw a ray with endpoint *T*.

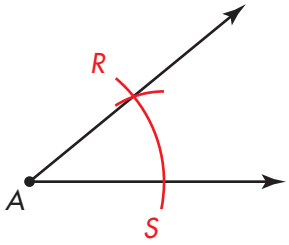
C Draw an arc with endpoint *S*.

D Draw an arc with endpoint *T*.

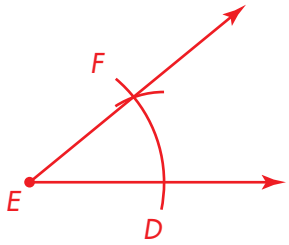
Set A, pages 452–453

Construct an angle congruent to angle *A*.

Draw a ray with endpoint *E*. With point *A* as center, use your compass to draw an arc that intersects both sides of angle *A*. Label the points of intersection *R* and *S*. With the same compass setting, use point *E* as center and draw an arc that intersects the ray at point *D*.



Open the compass to length *RS*. Then, with point *D* as center, draw an intersecting arc. Label the intersection *F*. Use a straightedge to draw ray *EF*.

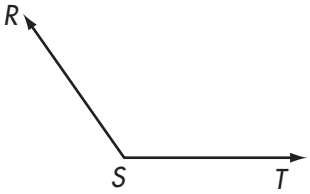


Angle *E* is congruent to angle *A*.

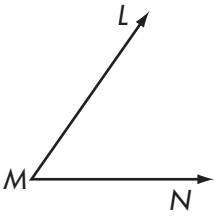
Remember that a construction uses only a compass and a straightedge.

Construct an angle congruent to the given angle.

1.



2.



Set B, pages 454-456

Construct a line perpendicular to another line.

Draw line *AB*.

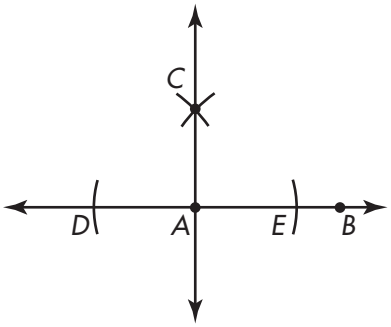
With *A* as center, draw two arcs that intersect line *AB*. Label the points of intersection *D* and *E*.

Open the compass wider.

Using *D* and *E* as centers, draw arcs that intersect. Label the intersection *C*.

Draw Line *CA*.

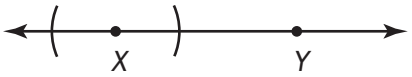
Line *CA* is perpendicular to line *AB*.



Remember to keep the compass open to the same setting when drawing the arcs with *D* and *E* as the centers.

1. Construct a line perpendicular to another line.

2. For the construction started below, what is the next step to construct a line perpendicular to line *XY*?



Set C, pages 458–460

Construct a triangle congruent to triangle *DEF*.

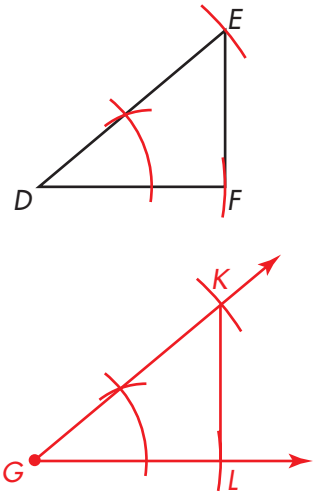
Draw a ray with endpoint *G*.

Construct angle *G*. Congruent to angle *D*.

Construct segments *GK* and *GL* on the sides of angle *G* so that they are congruent to segments *DE* and *DF*.

Draw segment *KL*.

Triangle *GKL* is congruent to triangle *DEF*.



Construct a rectangle *ABCD*.

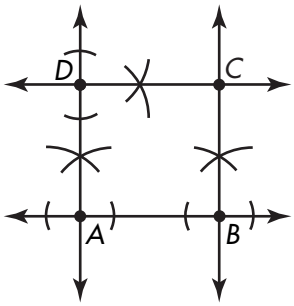
Draw line *AB*.

Construct line *DA* perpendicular to line *AB* at *A*.

Construct line *DC* perpendicular to line *DA* at *D*.

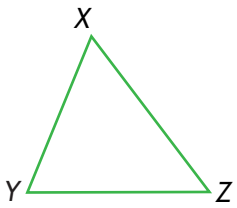
Construct line *CB* perpendicular to line *AB* at *B*.

Lines *DC* and *CB* intersect at *C*.



Remember to use the compass to measure the length of the original line segments by placing the point on one endpoint and the slider on the other endpoint.

1. Construct a triangle congruent to triangle *XYZ*.



2. Construct a rectangle *KLMN*.

Set D, pages 462–463

When you use objects to solve problems, follow these steps.

Step 1 Choose objects that can best model what is described in the problem.

Step 2 Use the objects to make a model of what you know.

Step 3 Use the objects to act out the action in the problem. Look for patterns.

Step 4 Find the answer in your model.

Remember to state clearly at the beginning what your objects represent in the problem.

1. How many total bricks are needed if the pattern extends to 4 bricks in the middle row?